

Quiz Answer

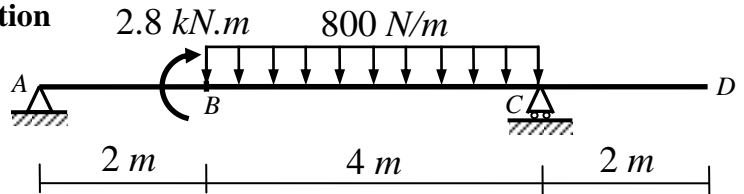
Question (1): (10 Marks)

For the shown beam, using the **double integration method**, determine:

- the deflection at B
- the deflection at D
- the slope at D

and sketch the elastic curve of the beam.

$$EI = 0.45 \times 10^6 \text{ N.m}^2$$



Solution:

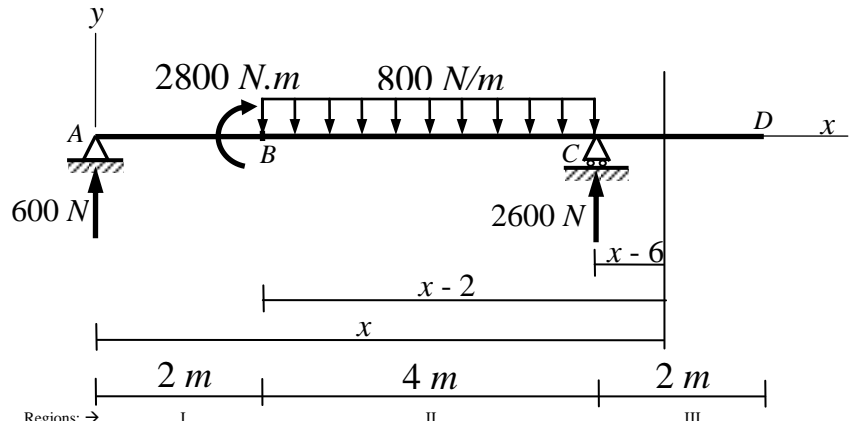
Reactions:

$$A_y(6) + 2800 - 800 \times 4(2) = 0$$

$$\rightarrow A_y = 600 \text{ N}$$

$$600 + C_y - 800 \times 4 = 0$$

$$\rightarrow C_y = 2600 \text{ N}$$



$$M = 600x \Big|_I + 2800(x-2)^0 - 800(x-2)^2/2 \Big|_II + 2600(x-6) + 800(x-6)^2/2 \Big|_III$$

$$EI y'' = 600x \Big|_I + 2800(x-2)^0 - 400(x-2)^2 \Big|_II + 2600(x-6) + 400(x-6)^2 \Big|_III$$

$$EI y' = 300x^2 \Big|_I + 2800(x-2) - 400(x-2)^3/3 \Big|_II + 1300(x-6)^2 + 400(x-6)^3/3 \Big|_III + C_1$$

$$EI y = 100x^3 \Big|_I + 1400(x-2)^2 - 100(x-2)^4/3 \Big|_II + 1300(x-6)^3/3 + 100(x-6)^4/3 \Big|_III + C_1 x + C_2$$

Boundary Conditions:

At $x = 0$, $y = 0 \rightarrow C_2 = 0$

At $x = 6 \text{ m}$, $y = 0 \rightarrow 0 = 100(6)^3 + 1400(4)^2 - 100(4)^4/3 + 6C_1 \rightarrow C_1 = -5911.11 \text{ N.m}^3$

So, the general equation of the deflection y at any distance x is,

$$EI y = 100x^3 \Big|_I + 1400(x-2)^2 - 100(x-2)^4/3 \Big|_II + 1300(x-6)^3/3 + 100(x-6)^4/3 \Big|_III - 5911.11x$$

(a) the deflection at B ($x=2$): in Region I:

$$EI y_B = 100(2)^3 - 5911.11(2) = -11022.22$$

$$y_B = -11022.22 / 450000 = -0.0245 \text{ m} = -24.5 \text{ mm} = \boxed{24.5 \text{ mm} \downarrow}$$

(b) the deflection at D ($x=8$): in Region III:

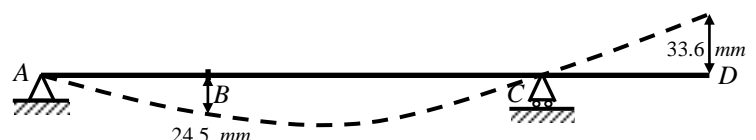
$$EI y_D = 100(8)^3 + 1400(6)^2 - 100(6)^4/3 + 1300(2)^3/3 + 100(2)^4/3 - 5911.11(8) = +15111.12$$

$$y_D = -15111.12 / 450000 = -0.03358 \text{ m} = +33.6 \text{ mm} = \boxed{33.6 \text{ mm} \uparrow}$$

(c) the slope at D ($x=8$): in Region III:

$$EI y'_D = 300(8)^2 + 2800(6) - 400(6)^3/3 + 1300(2)^2 + 400(2)^3/3 - 5911.11 = +7555.56$$

$$\theta_D = y'_D = +7555.56 / 450000 = \boxed{+0.0168 \text{ rad} = +0.96^\circ}$$



Elastic curve