Ministry of Higher Education
Giza Higher Institute of Engineering & Technology
Civil Engineering Department
Course Name: **Theory of Structures (3)**
Course Code : **CIV 301** Date : **30 / 12 / 2017**

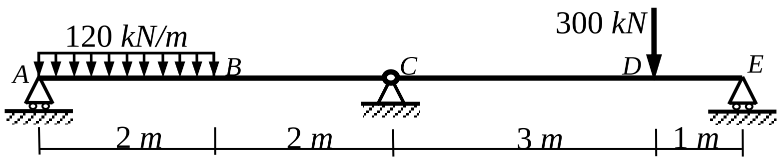
Academic Year : **2017/2018**
Semester : **First**
Level : **3rd**
Time : 3 **Hours**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Final Exam

Total Marks: **60** No. of Questions:**5** (Attempt all questions)

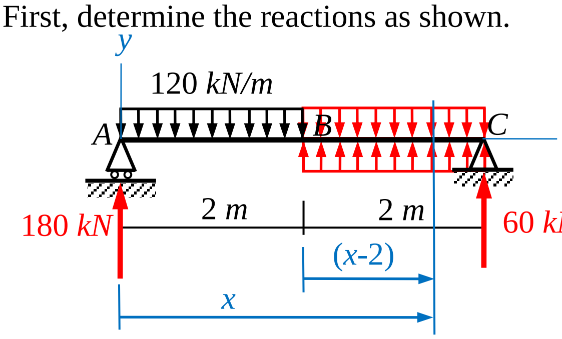
Question (1): (12 Marks)

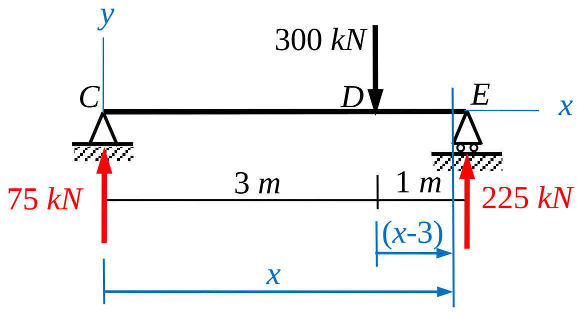
For the shown beam, using the **double integration method**:
(a) Determine the deflections at *B* and the mid-span *CE*.
(b) Determine the slopes just to the left and the right of *C*.
(c) Sketch the elastic curve of the beam.
$EI = 2\times10^7\ N.m^2$



Solution:

- Note that there is an intermediate hinge above the intermediate support at *C*. This intermediate hinge separates the beam to two parts (*AC* and *CE*) as shown.
- First, determine the reactions as shown.





$$EIy'' = M$$

$$EIy'' = 180x - 120x^2/2 + 120(x-2)^2/2$$

$$EIy' = 90x^2 - 20x^3 + 20(x-2)^3 + C_1$$

$$EIy = 30x^3 - 5x^4 + 5(x-2)^4 + C_1x + C_2$$

Boundary Conditions:

At $x = 0$, $y = 0$ → $C_2 = 0$

At $x = 4$, $y = 0$ →

$0 = 30(4)^3 - 5(4)^4 + 5(2)^4 + C_1(4) + 0$ → $C_1 = -180$

So, the general equation of the deflection y and slope $y'(\theta)$ at any distance x is,

$$EIy' = 90x^2 - 20x^3 + 20(x-2)^3 - 180$$

$$EIy = 30x^3 - 5x^4 + 5(x-2)^4 - 180x$$

The deflection at *B* is (at $x = 2\ m$)

$EI\delta_B = 30(2)^3 - 5(2)^4 - 180(2) = -200$

$\delta_B = -200/20000 = -0.01\ m$

$\delta_B = 10\ mm \downarrow$

The slope at section just to the left of *C* is (at $x = 4m$)

$EIy' = 90(4)^2 - 20(4)^3 + 20(2)^3 - 180 = 140$

$\theta_{c\ left} = 140/20000 = 0.007\ rad$

$\theta_{c\ left} = 0.401^o$ ↺

$$EIy'' = M$$

$$EIy'' = 75x - 300(x-3)$$

$$EIy' = 37.5x^2 - 150(x-3)^2 + C_3$$

$$EIy = 12.5x^3 - 50(x-3)^3 + C_3x + C_4$$

Boundary Conditions:

At $x = 0$, $y = 0$ → $C_4 = 0$

At $x = 4$, $y = 0$ →

$0 = 12.5(4)^3 - 50(1)^4 + C_3(4) + 0$ → $C_3 = -187.5$

So, the general equation of the deflection y and slope $y'(\theta)$ at any distance x is,

$$EIy' = 37.5x^2 - 150(x-3)^2 - 187.5$$

$$EIy = 12.5x^3 - 50(x-3)^3 - 187.5x$$

The deflection at *mid-span* is (at $x = 2\ m$)

$EI\delta_{mid-span} = 12.5(2)^3 - 187.5(2) = -275$

$\delta_{mid-span} = -275/20000 = -0.01375\ m$

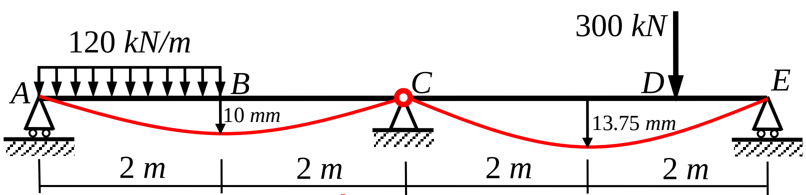
$\delta_{mid-span} = 13.75\ mm \downarrow$

The slope at section just to the right of *C* is (at $x = 0$)

$EIy' = 37.5(0)^2 - 187.5 = -187.5$

$\theta_{c\ right} = -187.5/20000 = -0.0094\ rad$

$\theta_{c\ right} = 0.537^o$ ↻



Elastic Curve

With my best wishes
Dr. M. Abdel-Kader

Question (2): (12 Marks)

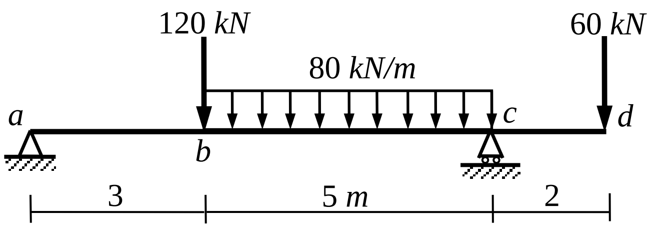
For the shown beam, using the **moment-area method**:

(a) Determine the slope at *a*.

(b) Determine the deflections at *b* and *d*.

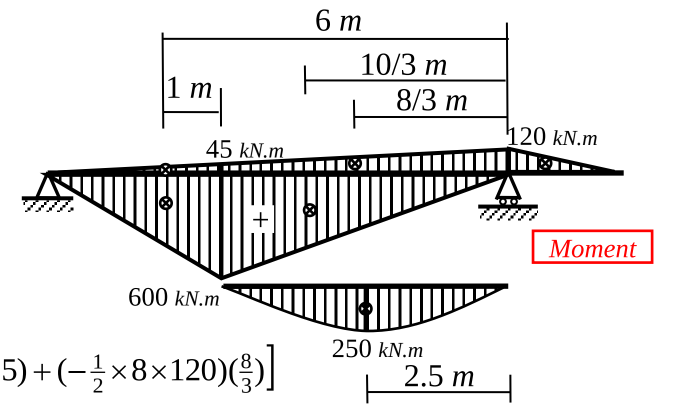
(c) Sketch the elastic curve of the beam.

$EI = 150\ MN.m^2$



Solution:

The bending moment diagram may be drawn as shown.



(a) The slope at *a*

$$\theta_a = \frac{t_{c/a}}{8}$$

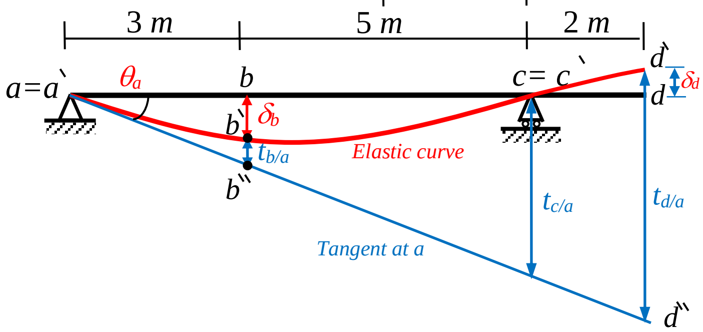
Apply the second moment-area theorem, then

$$t_{c/a} = \frac{1}{EI}\left[Area_{ac}.\overline{X}_c\right]$$

$$= \frac{1}{EI}\left[(\tfrac{1}{2}\times3\times600)(6)+(\tfrac{1}{2}\times5\times600)(\tfrac{10}{3})+(\tfrac{2}{3}\times5\times250)(2.5)+(-\tfrac{1}{2}\times8\times120)(\tfrac{8}{3})\right]$$

$$= \frac{33610}{3EI} = \frac{33610}{3\times150000} = 3351/45000 = 0.07469\ m$$

$$\therefore\ \theta_a = \frac{t_{c/a}}{8} = \frac{0.07469}{8} = 0.009336\ rad = 0.535^o$$



$\theta_a = 0.535^o$ ↻

(b) The deflection at *b*

The deflection at $b = \delta_b = bb'' - b'b'' = (3/8)\ t_{c/a} - t_{b/a}$

Applying the second moment-area theorem, then

$$t_{b/a} = \frac{1}{EI}\left[\text{First moment of area of M - diagram between } a \text{ and } b \text{ about } b\right]$$

$$= \frac{1}{EI}\left[Area_{ab}.\overline{X}_b\right] = \frac{1}{EI}\left[(\tfrac{1}{2}\times3\times600)(1)+(-\tfrac{1}{2}\times3\times45)(1)\right] = \frac{3330}{4EI} = \frac{3330}{4\times150000} = 0.00555\ m$$

$$\therefore\ \delta_b = bb'' - b'b'' = (3/8)\ t_{c/a} - t_{b/a} = (3/8)(\ 0.07469) - 0.00555 = 0.02246\ m$$

$\delta_b = 22.46\ mm$ ↓

(c) The deflection at *d*

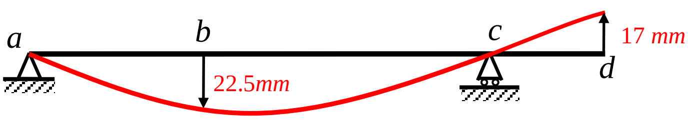
$$t_{d/a} = \frac{1}{EI}\left[Area_{ad}.\overline{X}_d\right]$$

$$= \frac{1}{EI}\left[(\tfrac{1}{2}\times3\times600)(8)+(\tfrac{1}{2}\times5\times600)(\tfrac{16}{3})+(\tfrac{2}{3}\times5\times250)(4.5)+(-\tfrac{1}{2}\times8\times120)(\tfrac{14}{3})+(-\tfrac{1}{2}\times2\times120)(\tfrac{4}{3})\right]$$

$$= \frac{16550}{EI} = \frac{16550}{150000} = 331/3000 = 0.110333\ m$$

$$\therefore\ \delta_d = d'd'' - dd'' = t_{d/a} - (10/8)\ t_{c/a} = 0.110333 - (10/8)(\ 0.07469) = 0.01697\ m$$

$\delta_d = 16.97\ mm$ ↑



Elastic curve

With my best wishes

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Question (3): (12 Marks)

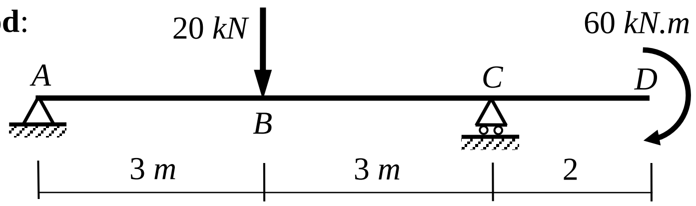
For the shown beam, using the **conjugate beam method**:

(a) Determine the slope at ***C***.

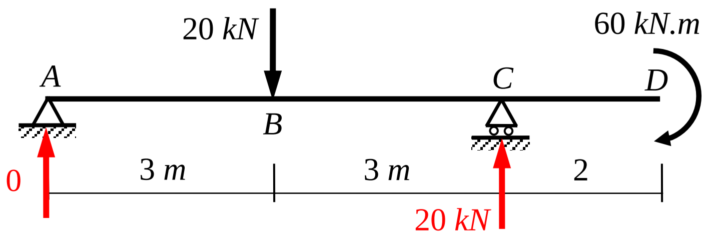
(b) Determine the deflections at *B* and ***D***.

(c) Sketch the elastic curve of the beam.

$EI = 40\times10^3\ kN.m^2$



Solution:

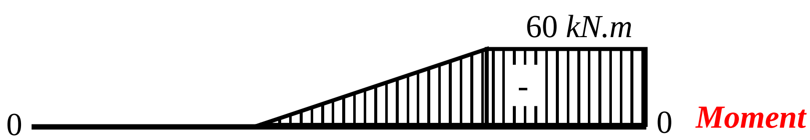


Reaction:

$+\circlearrowright \sum M_A = 0$

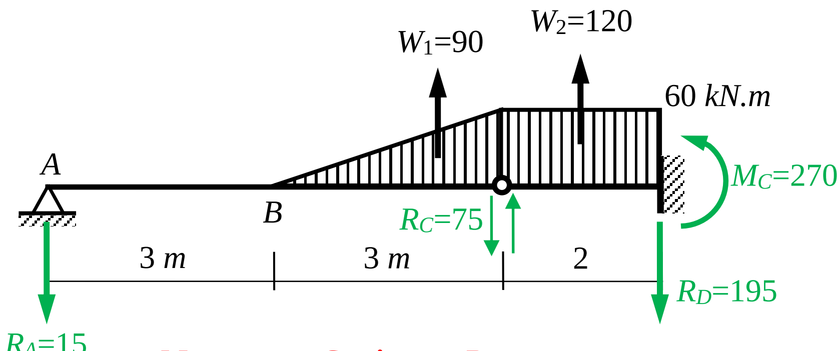
$A_y\,(6) - 20\times3 + 60 = 0 \rightarrow A_y = 0$

First construct the bending moment diagram of the real beam.



The resulting moment diagram is then loaded to the conjugate beam.

For the conjugate beam, determine the elastic reaction (R_C, M_B and M_D).



Moment on Conjugate Beam

$W_1 = \frac{1}{2}\times3\times60 = 90\ kN.m^2$

$W_2 = 2\times60 = 120\ kN.m^2$

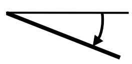
M_C for left part $= 0$

$R_A\,(6) - W_1\,(1) = 0 \rightarrow R_A = 15\ kN.m^2 \quad \rightarrow R_C = 90 - 15 = 75\ kN.m^2$

$M_B = -R_A\,(3) = -15(3) = -45\ kN.m^3$

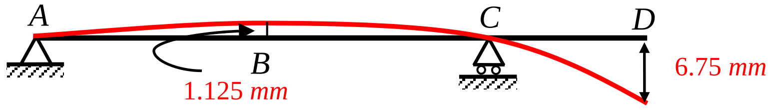
$M_D = R_C\,(2) + W_2(1) = 75(2) + 120(1) = 270\ kN.m^3$

(a) Slope at C = $R_C / EI = 75 / 40000 = 0.001875$ rad $= 0.1074^o$ ∴ $\theta_c = 0.11^o$



(b) Deflection at *B* = $M_B / EI = -45 / 40000 = -0.001125\ m = -1.125\ mm$ ∴ $\delta_B = 1.125\ mm$ ⬆

Deflection at *D* = $M_D / EI = 270 / 40000 = 0.00675\ m = 6.75\ mm$ ∴ $\delta_D = 6.75\ mm$ ⬇



Elastic Curve

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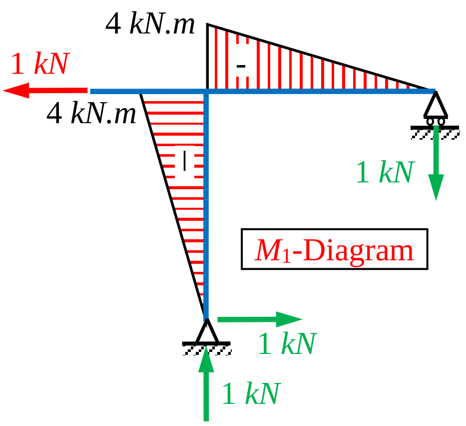
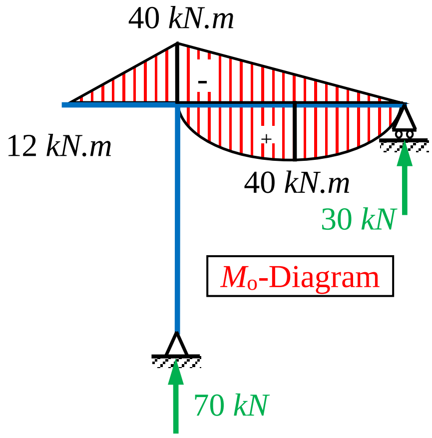
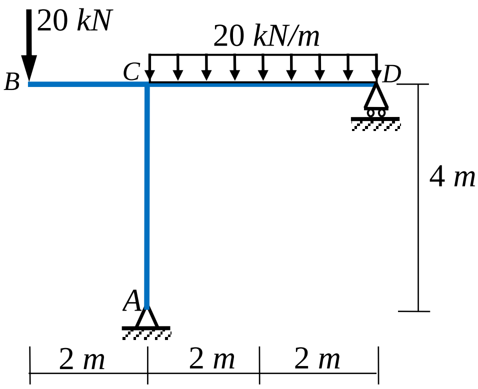
Question (4): (12 Marks)

For the shown frame and truss, using the **virtual work method**, determine the horizontal displacements at ***B*** (δ_{Bh}). For the frame, $EI = 20\times10^3$ $kN.m^2$. For the truss, assume that all members have the same axial rigidity $EA = 30000$ kN.

Solution:

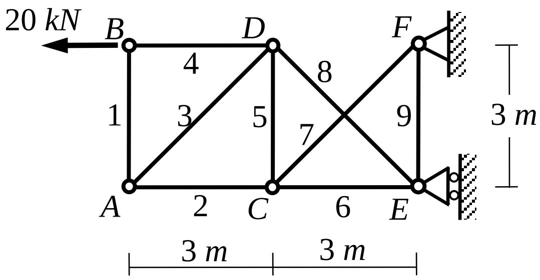


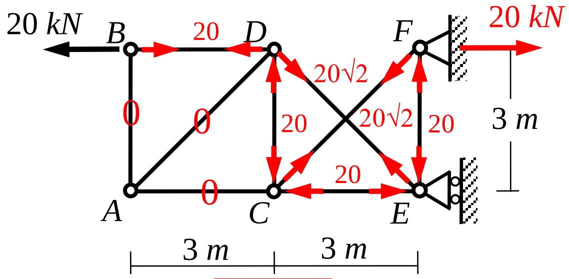




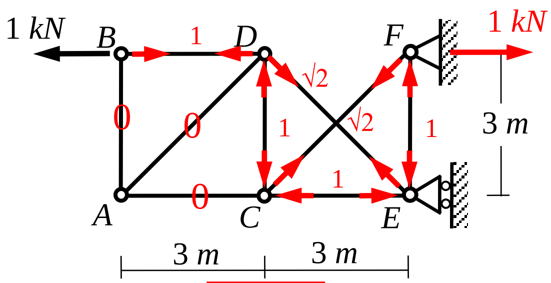
$$\delta_{Bh} = \int \frac{M_o M_1}{EI} dL = \frac{1}{EI}\left[\left(-\tfrac{1}{2}\times4\times40\right)\left(-\tfrac{2}{3}\times4\right)+\left(\tfrac{2}{3}\times4\times40\right)\left(-\tfrac{1}{2}\times4\right)\right]=0$$

$\therefore$ $\boxed{\delta_{Bh} = 0}$





N_o in kN



N_1 in kN

$$\delta_{Bh} = \Sigma\frac{N_0 N_1 L}{EA}$$

$$= \frac{4(20\times1\times3)+2(20\sqrt{2}\times\sqrt{2}\times3\sqrt{2})}{30000} = 0.01931\ m = 19.31\ mm$$

$\therefore$ $\boxed{\delta_{Bh} = 19.31\ mm \leftarrow}$

With my best wishes

Dr. M. Abdel-Kader

Question (5): (12 Marks)

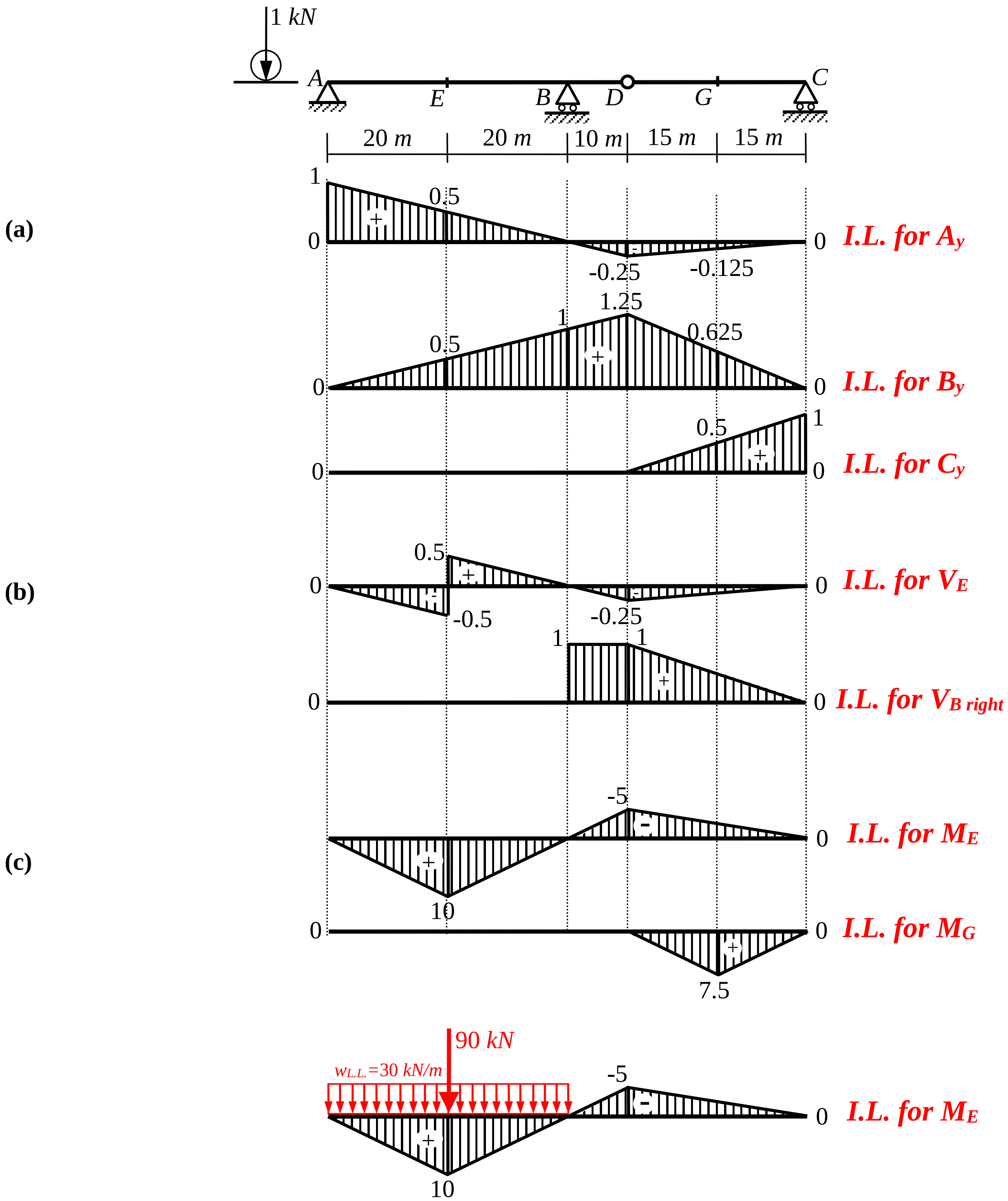
For the shown beam, draw the influence line for:

(a) The reactions A_y, B_y and C_y.

(b) The shear forces at the sections E and B_{right} .

(c) The bending moments at the sections E and G.

Also, determine the maximum moment at E caused by a concentrated moving load of 90 kN and a uniform live load of 30 kN/m.



For concentrated moving load, $M_{E\,max} = (90)(10) = 900\ kN.m$

For uniform live load, $M_{E\,max} = (0.5 \times 40 \times 10)(30) = 6000\ kN.m$

Total maximum moment at E, $M_{E\,max} = 900 + 6000 =$ **6900 $kN.m$**

With my best wishes

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