Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (4)**
Course Code : **CIV 302**

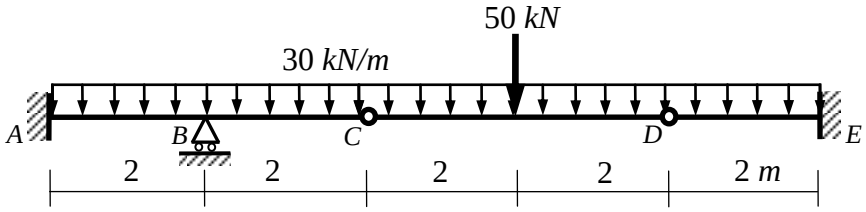
Academic Year : **2016-2017**
Semester : **Second**
Level : **3rd**
Time : **1½ Hours**
Date : **30 / 3 / 2017**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Mid-Term Exam

- The Exam consists of **2** questions in **1** page.

Question (1): (10 Marks)

For the shown beam, using the **3-M equation**,



(a) draw the shear force and bending moment diagrams due to the given loads.
(b) calculate the moment at fixed support *A* due to settlement of support *B* by an amount of 1 *cm*.
$EI = 40000\ kN.m^2$

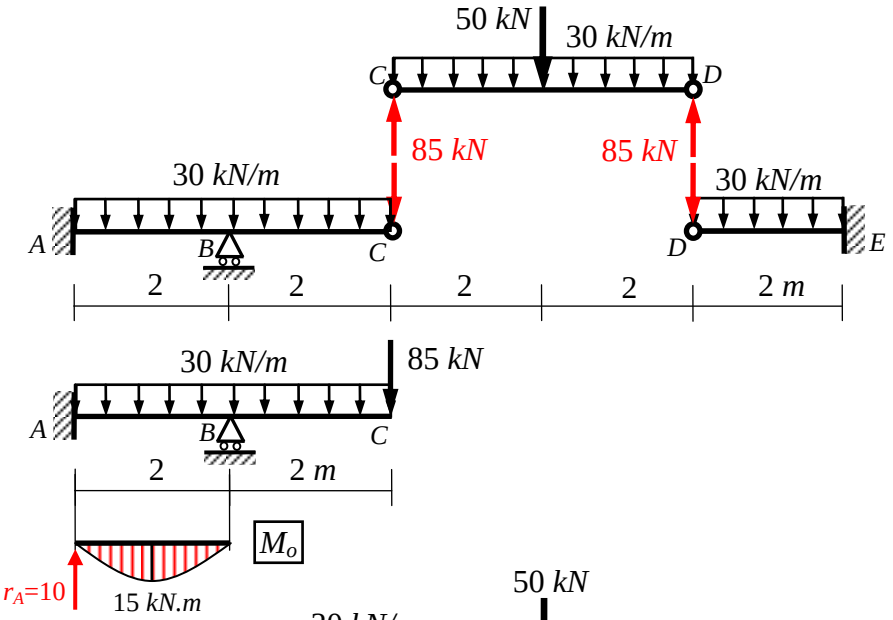
Answer:

(a) Separate the beam as shown

Only the part *ABC* is statically indeterminate.

The moment at *B*, M_B = -230 *kN.m*

The simply supported moment diagram on *AB* is as shown and the elastic reaction at *A* =10 $kN.m^2$.



Applying three-moment equation at *A* (for spans A_oA and *AB*):

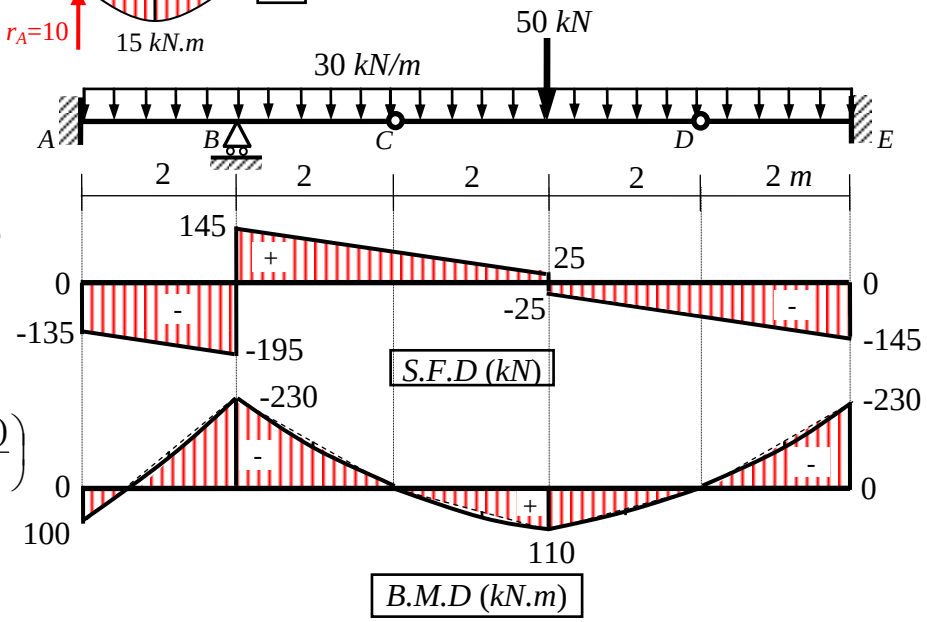
$$2M_A(2) + M_B(2) = -6r_A = -6(10)$$

$$4M_A + 2M_B = -60$$

$$2M_A - 230 = -30$$

→ M_A = +100 *kN.m*

The bending moment and shear force diagram are shown above.



(b) Applying the 3-M equation at *A*:
(note that $\Delta_A = 0$ and $\Delta_B = 1\ cm = 0.01\ m$)

$$2M_A(2) + M_B(2) = 6EI\left(\frac{(0-0)}{2} + \frac{(0-0.1)}{2}\right)$$

$$4M_A + 0 = 6EI\left(\frac{-0.01}{2}\right) = -0.03EI$$

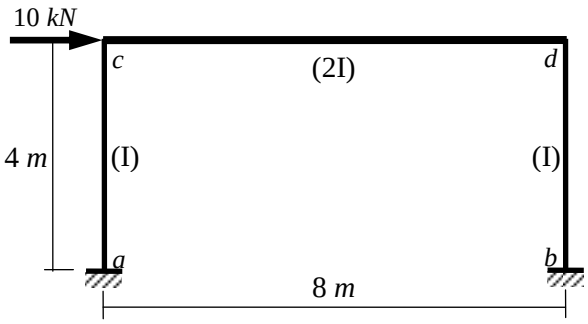
$$M_A = -\frac{0.03 \times 40000}{4} = -300\ kN.m$$

M_A due to settlement of support *B* (by 1 *cm*) = -300 *kN.m*

Please turn over

Question (2): (10 Marks)

For the shown frame, using the **Consistent Deformations** (**Virtual Work**) method, draw the bending moment diagram due to the applied load. Note that the relative moments of inertia are given between brackets as shown and E is constant.



Answer:

$$\delta_{10} = \frac{1}{EI}\left[\left(\tfrac{1}{2}\times 4\times 40\right)\left(\tfrac{4}{3}\right)\right] = 320/3EI$$

$$\delta_{20} = \frac{1}{EI}\left[\left(-\tfrac{1}{2}\times 4\times 40\right)(8)\right] = -640/EI$$

$$\delta_{30} = \frac{1}{EI}\left[\left(\tfrac{1}{2}\times 4\times 40\right)(1)\right] = 80/EI$$

$$\delta_{11} = \frac{1}{EI}\left[2\left(\tfrac{1}{2}\times 4\times 4\right)\left(\tfrac{2}{3}\times 4\right)\right] + \frac{1}{2EI}\left[(8\times 4)(4)\right] = 320/3EI$$

$$\delta_{22} = \frac{1}{EI}\left[(4\times 8)(8)\right] + \frac{1}{2EI}\left[\left(\tfrac{1}{2}\times 8\times 8\right)\left(\tfrac{2}{3}\times 8\right)\right] = 1024/3EI$$

$$\delta_{33} = \frac{1}{EI}\left[2(4\times 1)(1)\right] + \frac{1}{2EI}\left[(8\times 1)(1)\right] = 12/EI$$

$$\delta_{12} = \delta_{21} = \frac{1}{EI}\left[(-4\times 8)(2)\right] + \frac{1}{2EI}\left[\left(-\tfrac{1}{2}\times 8\times 8\right)(4)\right] = -128/EI$$

$$\delta_{13} = \delta_{31} = \frac{1}{EI}\left[2\left(\tfrac{1}{2}\times 4\times 4\right)(1)\right] + \frac{1}{2EI}\left[(8\times 4)(1)\right] = 32/EI$$

$$\delta_{23} = \delta_{32} = \frac{1}{EI}\left[(4\times 8)(1)\right] + \frac{1}{2EI}\left[\left(\tfrac{1}{2}\times 8\times 8\right)(1)\right] = -48/EI$$

$$320/3 + (320/3)X_1 - 128\ X_2 + 32\ X_3 = 0 \quad \text{--- (1)}$$

$$-640 - 128\ X_1 - (1024/3)\ X_2 - 48\ X_3 = 0 \quad \text{--- (2)}$$

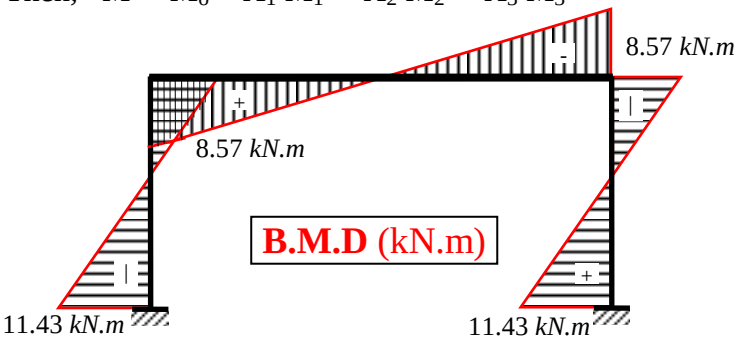
$$\times 4 \left\{ \begin{array}{l} 80 + 32\ X_1 - 48\ X_2 + 12\ X_3 = 0 \quad \text{--- (3)} \\ 320 + 128\ X_1 - 192\ X_2 + 48\ X_3 = 0 \quad \text{--- (3')} \end{array} \right.$$

From Eqs. (2) + (3') → $X_2 = 15/7 = 2.1429\ kN \uparrow$

Then, from Eq. (1)×3 and Eq. (2)×2 → $X_1 = 5\ kN \leftarrow$ and

$X_3 = -80/7 = -11.429\ kN \circlearrowright = 11.43\ kN \circlearrowleft$

Then, $M = M_o + X_1 M_1 + X_2 M_2 + X_3 M_3$

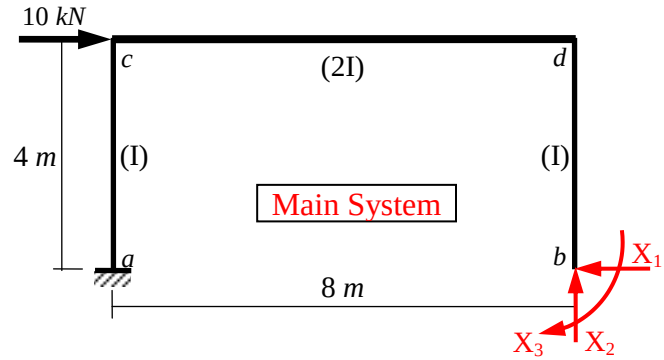


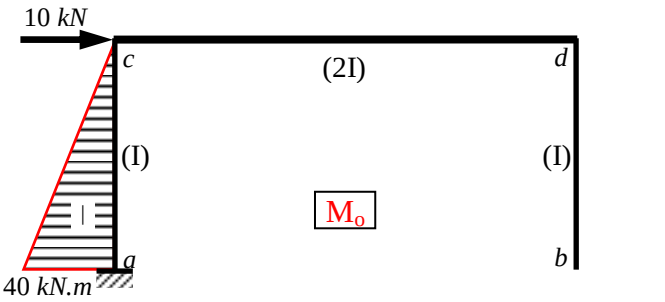
$M_a = -40 + 5\ (0) + (15/7)\ (8) + (-80/7)(-1) = -11.429\ kN.m$

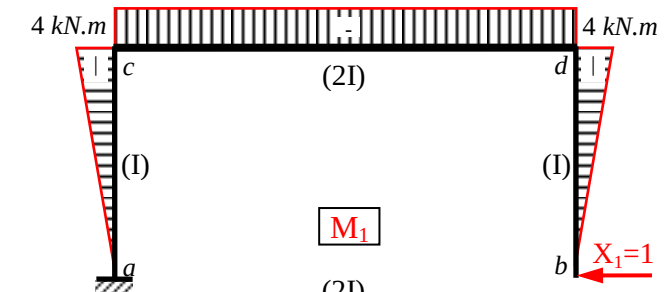
$M_b = 0 + 5\ (-4) + (15/7)\ (8) + (-80/7)(-1) = 8.5714\ kN.m$

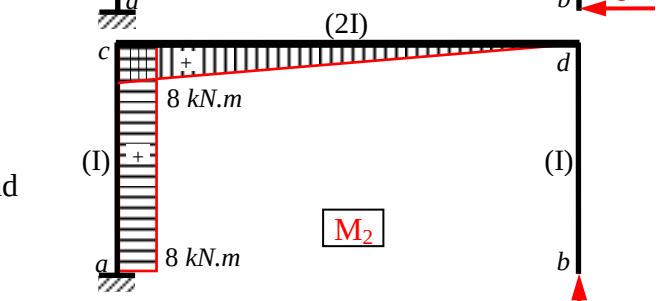
$M_c = 0 + 5\ (-4) + (15/7)\ (0) + (-80/7)(-1) = -8.5714\ kN.m$

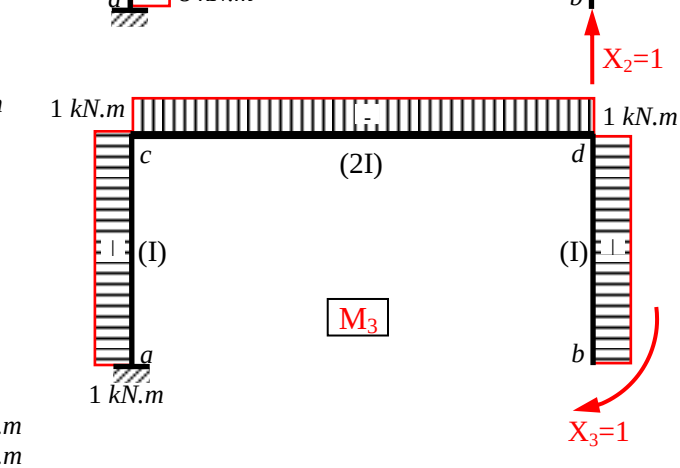
$M_d = 0 + 5\ (0) + (15/7)\ (0) + (-80/7)(-1) = 11.429\ kN.m$











With my best wishes

Dr. M. Abdel-Kader