Ministry of Higher Education
Giza Higher Institute of Engineering & Technology
Civil Engineering Department
Course Name: **Theory of Structures (3)**
Course Code : **CIV 301** Date : **13 / 11 / 2017**

Academic Year : **2017/2018**
Semester : **First**
Level : **3rd**
Time : **1¼ Hours**
Examiner: ***Dr. M. Abdel-Kader***

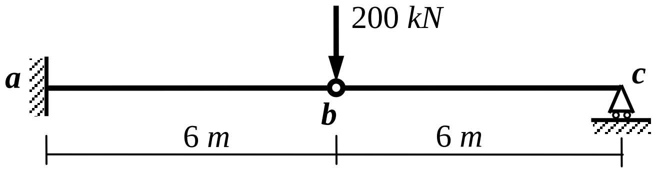
Answer of Mid-Term Exam

Total Marks: **20** No. of Questions:**2** (Attempt all questions)

Name: **Code:**

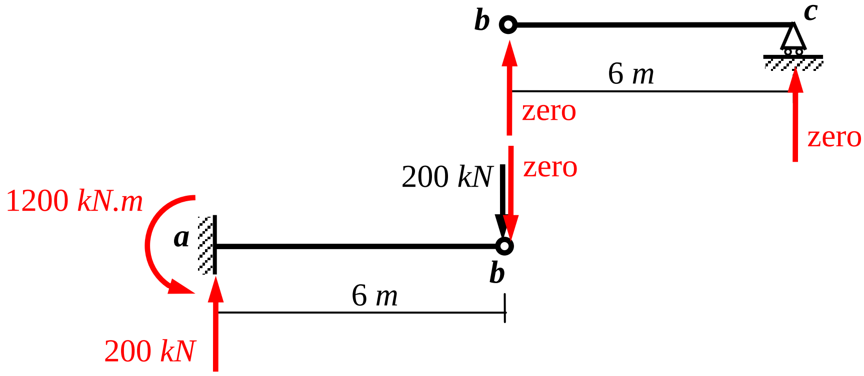
Question (1): (10 Marks)

Using the **double integration method**, determine the deflection at ***b***, the slope at section just to the left of ***b***, and sketch the elastic curve of the beam.
$EI = 6\times10^5\ kN.m^2$



Solution:

- First, determine the reactions as shown.
- Note that all the required (y_b and y'_{bleft}) are in the region *a-b* and no reaction comes form part *b-c* to part *a-b*, so only part *a-b* should be studied.



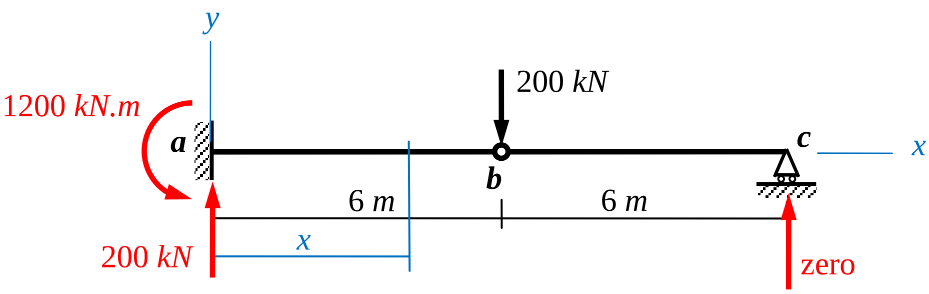
For region *a-b*

$$EIy'' = M$$

$$EIy'' = 200x - 1200$$

$$EIy' = 100x^2 - 1200x + C_1$$

$$EIy = \frac{100}{3}x^3 - 600x^2 + C_1 x + C_2$$



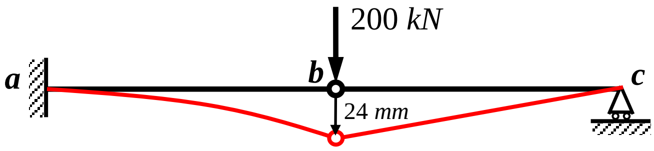
Boundary Conditions:

At $x = 0$, $y' = 0$ → $C_1 = 0$
At $x = 0$, $y = 0$ → $C_2 = 0$

So, the general equation of the deflection y and slope y' (θ) at any distance x is,

$$EIy = \frac{100}{3}x^3 - 600x^2$$

$$EIy' = 100x^2 - 1200x$$



Elastic curve

The deflection at ***b*** is at $x = 6\ m$

$$EI\delta_b = \frac{100}{3}(6)^3 - 600(6)^2 = -14400$$

$$\delta_b = -14400/600000 = -0.024\ m$$

$\delta_b = 24\ mm \downarrow$

The slope at section just to the left of ***b*** is at $x = 6\ m$

$$EIy' = 100(6)^2 - 1200(6) = -3600$$

$$\theta_b = -3600/600000 = -0.006\ rad$$

$\theta_b = 0.344^{o}$ ↻

With my best wishes
Dr. M. Abdel-Kader

Question (2): (10 Marks)

For the shown beam, using the **moment-area method**, determine:

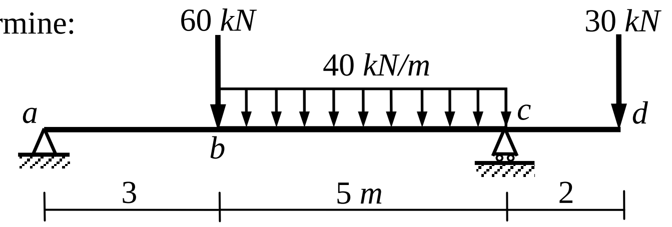
(a) The slope at a

(b) The deflection at b

(c) The deflection at d

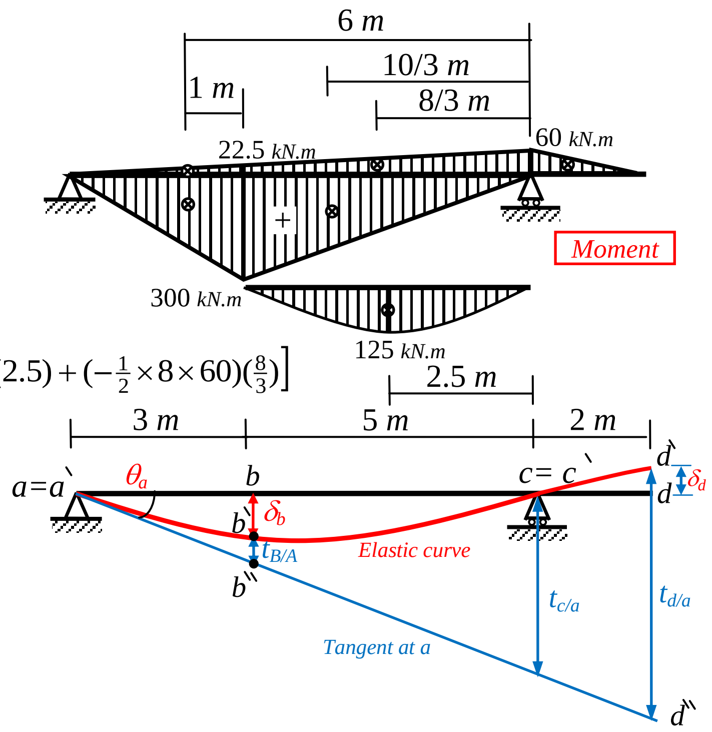
and sketch the elastic curve of the beam.

$EI = 60\ MN.m^2$



Solution:

The bending moment diagram may be drawn as shown.



(a) The slope at a

$$\theta_a = \frac{t_{c/a}}{8}$$

Apply the second moment-area theorem, then

$$t_{c/a} = \frac{1}{EI}\left[Area_{ac}.\bar{X}_c\right]$$

$$= \frac{1}{EI}\left[(\tfrac{1}{2}\times 3\times 300)(6) + (\tfrac{1}{2}\times 5\times 300)(\tfrac{10}{3}) + (\tfrac{2}{3}\times 5\times 125)(2.5) + (-\tfrac{1}{2}\times 8\times 60)(\tfrac{8}{3})\right]$$

$$= \frac{16805}{3EI} = \frac{16805}{3\times 60000} = 3351/36000 = 0.09336\ m$$

$$\therefore\ \theta_a = \frac{t_{c/a}}{8} = \frac{0.09336}{8} = 0.01167\ rad = 0.6686^o$$

$$\boxed{\theta_a = 0.67^o\ \circlearrowright}$$

(b) The deflection at b

The deflection at $b = \delta_b = bb'' - b'b'' = (3/8)\ t_{c/a} - t_{b/a}$

Applying the second moment-area theorem, then

$$t_{b/a} = \frac{1}{EI}\left[\text{First moment of area of M - diagram between } a \text{ and } b \text{ about } b\right]$$

$$= \frac{1}{EI}\left[Area_{ab}.\bar{X}_b\right] = \frac{1}{EI}\left[(\tfrac{1}{2}\times 3\times 300)(1) + (-\tfrac{1}{2}\times 3\times 22.5)(1)\right] = \frac{1665}{4EI} = \frac{1665}{4\times 60000} = 0.0069375\ m$$

$$\therefore\ \delta_b = bb'' - b'b'' = (3/8)\ t_{c/a} - t_{b/a} = (3/8)(\ 0.09336) - 0.0069375 = 0.0280725\ m$$

$$\boxed{\delta_b = 28.1\ mm \downarrow}$$

(c) The deflection at d

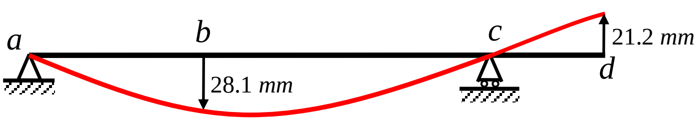
$$t_{d/a} = \frac{1}{EI}\left[Area_{ad}.\bar{X}_d\right]$$

$$= \frac{1}{EI}\left[(\tfrac{1}{2}\times 3\times 300)(8) + (\tfrac{1}{2}\times 5\times 300)(\tfrac{16}{3}) + (\tfrac{2}{3}\times 5\times 125)(4.5) + (-\tfrac{1}{2}\times 8\times 60)(\tfrac{14}{3}) + (-\tfrac{1}{2}\times 2\times 60)(\tfrac{4}{3})\right]$$

$$= \frac{8275}{EI} = \frac{8275}{60000} = 331/2400 = 0.1379167\ m$$

$$\therefore\ \delta_d = d'd'' - dd'' = t_{d/a} - (10/8)\ t_{c/a} = 0.1379167 - (10/8)(\ 0.09336) = 0.021217\ m$$

$$\boxed{\delta_d = 21.2\ mm \uparrow}$$



Elastic curve

With my best wishes

Dr. M. Abdel-Kader