Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (3)**
Course Code : **CIV 301**

Academic Year : **2015–2016**
Semester : **First**
Level : **3rd**
Time : **1½ Hours**
Date : **22 / 11 / 2015**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Mid-Term Exam

Question (1): (10 Marks)

For the shown beam, using the **moment-area method**, determine:

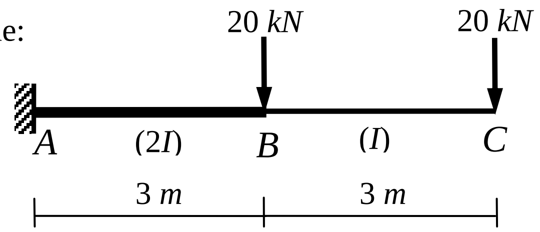
(a) the deflection at *B*

(b) the slope at *C*

(c) the deflection at *C*

and sketch the elastic curve of the beam.

$EI = 90.0\times10^6\ N.m^2$

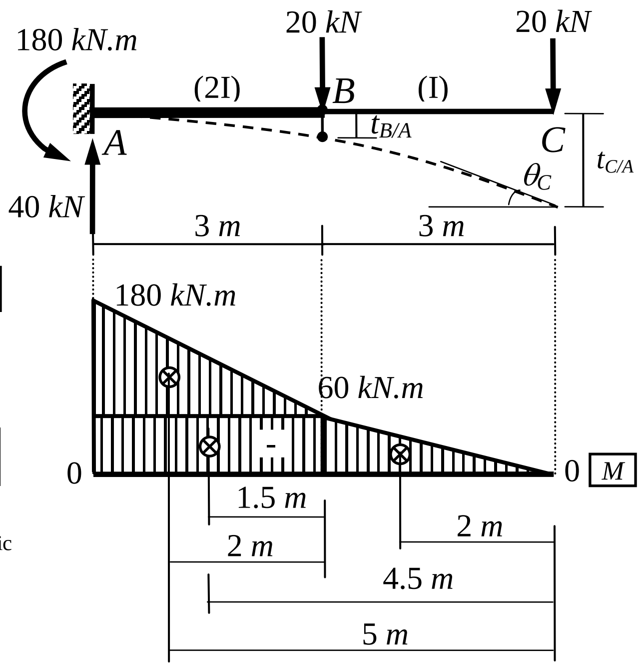


Solution:

The bending moment diagram may be drawn as shown.

(a) the deflection at B

The deflection at *B* is equal to the deviation of point *B* relative to the tangent of the elastic curve at point *A*, $t_{B/A}$. Applying the second moment-area theorem, then



$$\delta_B = t_{B/A}$$

$$= \frac{1}{EI}\left[\text{First moment of area of M - diagram between } A \text{ and } B \text{ about } B\right]$$

$$= \frac{1}{EI}\left[Area_{AB}.\bar{X}_B\right] = \frac{1}{2EI}\left[(-3\times60)(1.5)+(-\tfrac{1}{2}\times3\times120)(2)\right]$$

$$= -\frac{315}{EI} = -\frac{315}{90000} = \text{- } 0.0035\ m = 3.5\ mm \downarrow$$

$$\boxed{\delta_B = 3.5\ mm \downarrow}$$

(b) the slope at C

Since the slope at *A* (θ_A) is equal to zero, the change in slope between the tangents of the elastic curve at points *C* and *A* (θ_{CA}) is equal to the slope at *C* (θ_C),

$$\theta_{CA} = \theta_C - \theta_A = \theta_C - 0 = \theta_C$$

Apply the first moment-area theorem, then

$$\theta_C = \theta_{CA} = \frac{1}{EI}\left[\text{Area of M - diagram between the points } A \text{ and } C\right]$$

$$= \frac{1}{EI}\left[Area_{AC}\right] = \frac{1}{2EI}\left[(-3\times60)+(-\tfrac{1}{2}\times3\times120)\right]+\frac{1}{EI}\left[-\tfrac{1}{2}\times3\times60\right] = -\frac{270}{EI} = -\frac{270}{90000} = \text{- } 0.003\ rad$$

$$\boxed{\theta_C = 0.003\ rad = 0.172^o \circlearrowright}$$

(c) the deflection at C

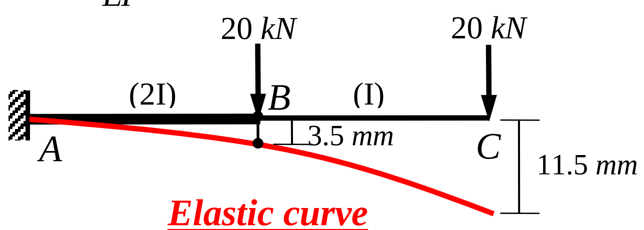
$$\delta_C = t_{C/A} = \frac{1}{EI}\left[\text{First moment of area of M - diagram between } A \text{ and } C \text{ about } C\right]$$

$$= \frac{1}{EI}\left[Area_{AC}.\bar{X}_C\right] = \frac{1}{2EI}\left[(-3\times60)(4.5)+(-\tfrac{1}{2}\times3\times120)(5)\right]+\frac{1}{EI}\left[(-\tfrac{1}{2}\times3\times60)(2)\right]$$

$$= -\frac{1035}{EI} = -\frac{1035}{90000} = \text{- } 0.0115\ m$$

$$= 11.5\ mm \downarrow$$

$$\boxed{\delta_C = 11.5\ mm \downarrow}$$



With my best wishes
Dr. M. Abdel-Kader

Question (2): (10 Marks)

For the shown beam, using the **conjugate beam method**, determine:

(a) the slope at A

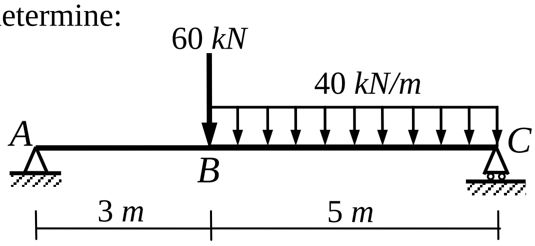
(b) the slope at B

(c) the deflection at B

and sketch the elastic curve of the beam.

$E = 200\ GPa$

$I = 290\times10^6\ mm^4$



Solution:

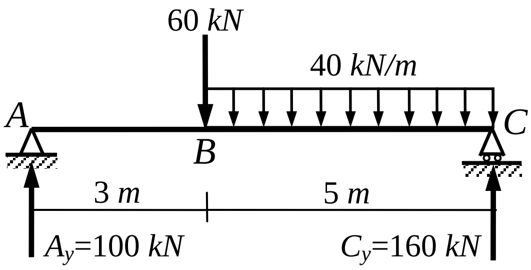
Reaction:

$+\circlearrowright\sum M_C = 0:$

$A_y(8) - 60(5) - 40\times5(2.5) = 0 \rightarrow A_y = 100\ kN \uparrow$

$+\uparrow\sum F_y = 0:$

$A_y + C_y - 60 - 40\times5 = 0 \rightarrow C_y = 160\ kN \uparrow$

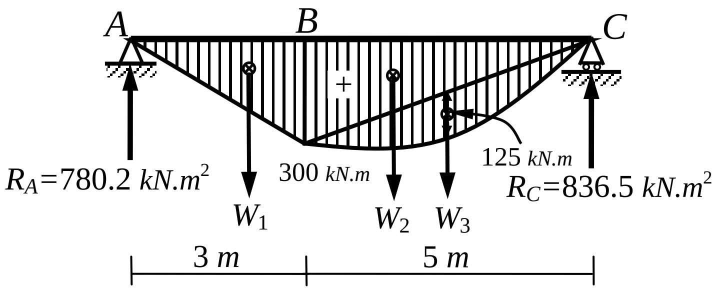


Construct the bending moment diagram of the real beam.
The resulting moment diagram is then loaded to the conjugate beam.
For the conjugate beam, determine the elastic reaction (R_A and R_C) at supports.

$W_1 = \frac{1}{2}\times3\times300 = 450\ kN.m^2$

$W_2 = \frac{1}{2}\times5\times300 = 750\ kN.m^2$

$W_3 = \frac{2}{3}\times5\times125 = 1250/3\ kN.m^2$



$+\circlearrowright\sum M_C = 0$

$R_A(8) - W_1(5+1) - W_2(2\times5/3) - W_3(5/2) = 0$

$8R_A = 450(6) + 750(10/3) + (1250/3)(2.5) \rightarrow R_A = 780.2\ kN.m^2$

$+\uparrow\sum F_y = 0 \rightarrow R_C = 836.5\ kN.m^2$

$E = 200\ GPa = 200\times10^6\ kN/m^2 \quad I = 290\times10^6\ mm^4 = 290\times10^{-6}\ m^4 \quad EI = 58000\ kN.m^2$

(a) the slope at *A*

$\theta_A = R_A/EI = 780.2/58000 = 0.0135\ rad = 0.77^\circ$

$\theta_A = 0.77^\circ \circlearrowright$

(b) the slope at *B*

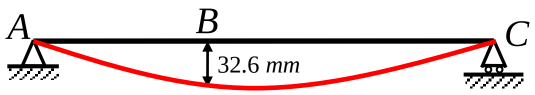
$\theta_B = Shear\ at\ B/EI = (R_A - W_1)/58000 = (780.2 - 450)/58000 = 0.0057\ rad = 0.33^\circ$

$\theta_B = 0.33^\circ \circlearrowright$

(c) the deflection at *B*

$\delta_B = Moment\ at\ B/EI = (R_A\times3 - W_1\times1)/58000 = 1890.6/58000 = 0.0326\ m = 32.6\ mm$

$\delta_B = 32.6\ mm \downarrow$



Elastic curve

With my best wishes

Dr. M. Abdel-Kader