Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (4)**
Course Code : **CIV 302**

Academic Year : **2015-2016**
Semester : **Second**
Level : **3rd**
Time : **1½ Hours**
Date : **5 / 4 / 2016**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Mid-Term Exam

Question (1): (12 Marks)

For the shown beam, using the **three-moment equation**,
(a) draw the shear force and bending moment diagrams due to the given loads.
(b) calculate the percentage increase in the moment at fixed support *C* due to settlement of support *B* by an amount of 10 *mm*.
$EI = 48000\ kN.m^2$

Solution:

(a) The simply supported moment diagram on *BC* is as shown.

The moment at *B*, M_B = -90 *kN.m*

Applying three-moment equation at C (for spans *BC* and CC_o):

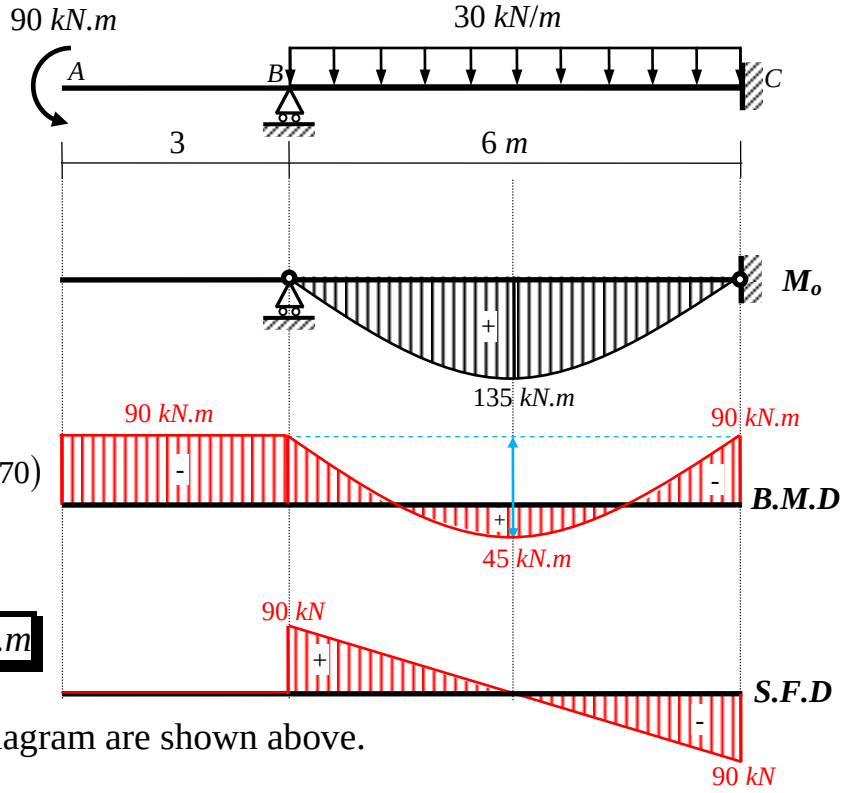
$$M_B(6)+2M_C(6)=-6\left(\frac{A_1\bar{x}_1}{L_1}\right)=-6r_C$$

$$M_B(6)+2M_C(6)=-6\left(\frac{(2/3\times6\times135)3}{6\times I}\right)=-6(270)$$

$$6M_B + 12M_C = -6(270)$$

$$M_B + 2M_C = -270$$

$$-90 + 2M_C = -270 \rightarrow \boxed{M_C = -90\ kN.m}$$



The bending moment and shear force diagram are shown above.

(b) Applying the three-moment equation at *C*: (note that $\Delta_C = 0$ and $\Delta_B = 10\ mm = 0.01\ m$)

$$M_B(6)+2M_C(6) = 6EI\left(\frac{(0-0.01)}{6}+\frac{(0-0)}{6}\right)$$

$$12M_C = 6EI\left(\frac{-0.01}{6}\right)=-0.01EI$$

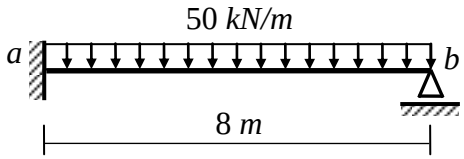
$$M_C = -\frac{0.01\times48000}{12}=-40\,kN.m$$

$\therefore$ Percentage increase in M_C = (40/90)×100 = 44.4 %

With my best wishes
Dr. M. Abdel-Kader

Question (2): (8 Marks)

For the shown beam, using the **Consistent Deformations** (**Virtual Work**) method, draw the bending moment diagram due to the given load.



Solution:

$$\delta_{10} = \int_a^b \frac{M_o M_1}{EI} dL = \frac{1}{EI}\left[\left(-\frac{1}{3}\times 8\times 1600\right)\left(\frac{3}{4}\times 8\right)\right] = \frac{-25600}{EI}$$

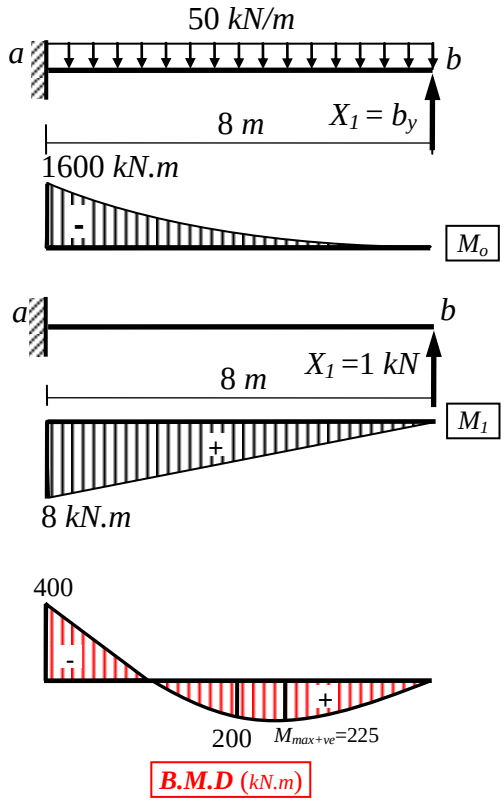
$$\boxed{\delta_{10} = \frac{-25600}{EI}}$$

$$\delta_{11} = \int_a^b \frac{M_1 M_1}{EI} dL = \frac{1}{EI}\left[\left(\frac{1}{2}\times 8\times 8\right)\left(\frac{2}{3}\times 8\right)\right] = \frac{512}{3EI}$$

$$\boxed{\delta_{11} = \frac{512}{3EI}}$$

$$\delta_{bv} = \delta_{10} + X_1\delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-25600\times 3}{512} = 150\ kN$$

$$\boxed{X_1 = b_y = 150\ kN \uparrow}$$



Another Solution:

$$\delta_{10} = \int_a^b \frac{M_o M_1}{EI} dL = \frac{1}{EI}\left[\left(\frac{2}{3}\times 8\times 400\right)\left(-\frac{1}{2}\times 1\right)\right] = \frac{-3200}{3EI}$$

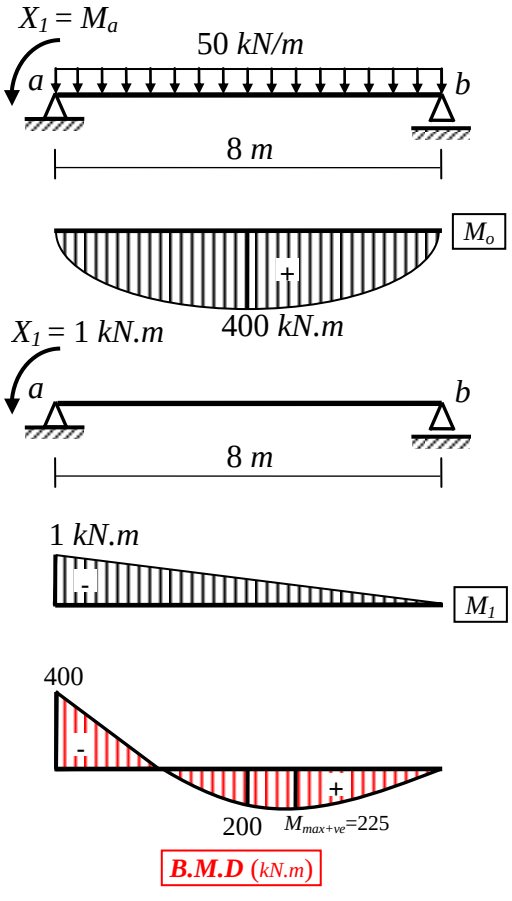
$$\boxed{\delta_{10} = \frac{-3200}{3EI}}$$

$$\delta_{11} = \int_a^b \frac{M_1 M_1}{EI} dL = \frac{1}{EI}\left[\left(-\frac{1}{2}\times 8\times 1\right)\left(-\frac{2}{3}\times 1\right)\right] = \frac{8}{3EI}$$

$$\boxed{\delta_{11} = \frac{8}{3EI}}$$

$$\theta_a = \delta_{10} + X_1\delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-3200}{8} = 400\ kN.m$$

$$\boxed{X_1 = M_a = 400\ kN.m \circlearrowleft}$$



With my best wishes

Dr. M. Abdel-Kader