Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (4)**
Course Code : **CIV 302**

Academic Year : **2014-2015**
Semester : **Second**
Level : **3rd**
Time : **1½ Hours**
Date : **29 / 3 / 2015**
Examiner: ***Dr. M. Abdel-Kader***

Solution of Mid-Term Exam

Question (1): (10 Marks)

Using the three-moments equation, draw the shear force and bending moment diagrams for the shown beam. The relative moments of inertia are given between brackets.

Solution

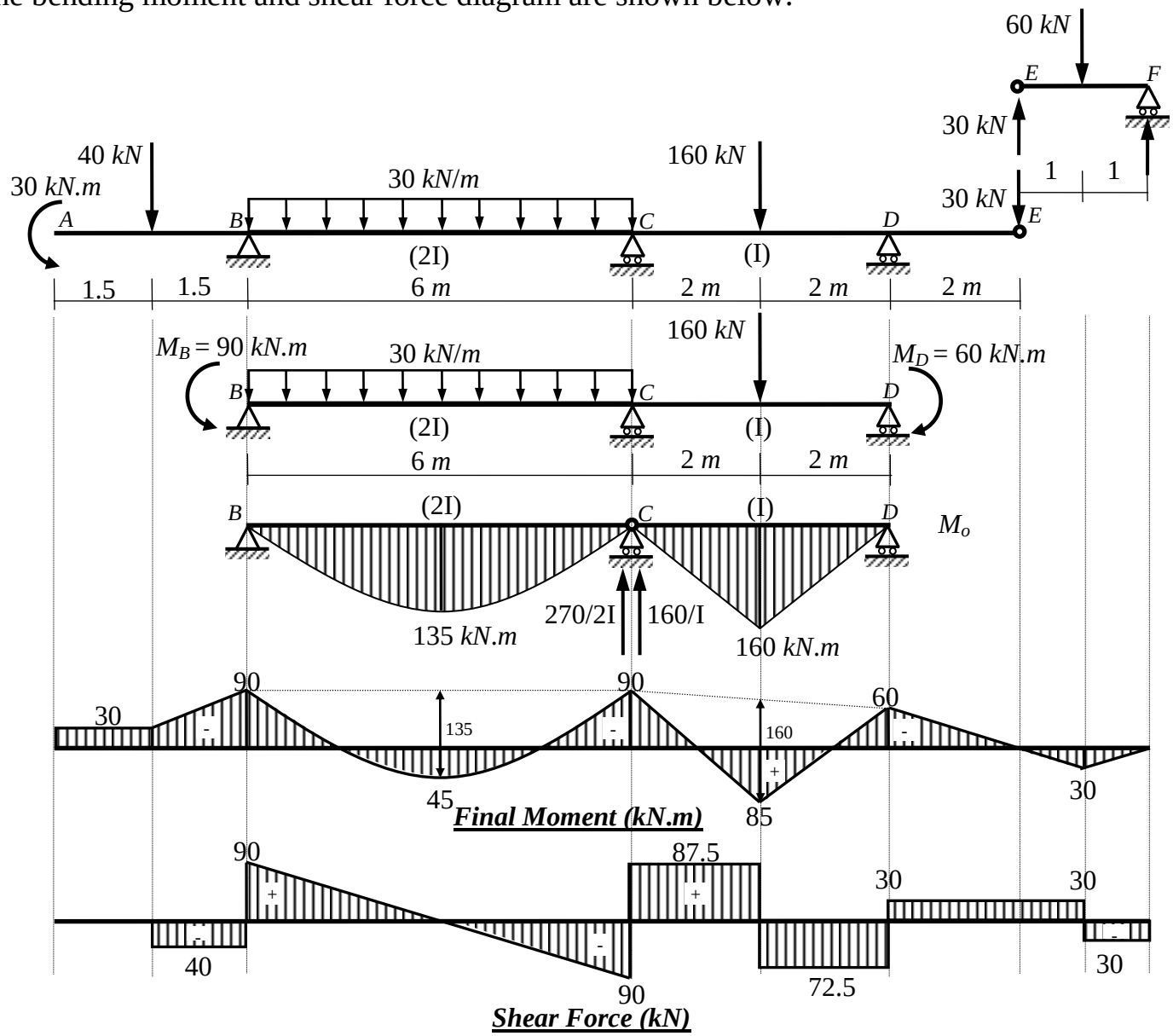
- The simply supported moment diagram (M_O) on spans *BC and CD* is shown below.
- The moment at *B*, M_B = -90 *kN.m* and the moment at *D*, M_D = -60 *kN.m*
- Applying three-moments equation for spans *BC* and *CD*:

$$M_B\left(\frac{6}{2I}\right)+2M_C\left(\frac{6}{2I}+\frac{4}{I}\right)+M_D\left(\frac{4}{I}\right)=-6\left(\frac{270}{2I}+\frac{160}{I}\right)$$

$$(-90)\,(3)+\quad 14M_C\quad +(-60)\,(4)=-6\,(295)$$

$$-510+14M_C=-1770 \quad\rightarrow\quad 14M_C=-1260 \quad\rightarrow\quad \boxed{M_C=-90\ kN.m}$$

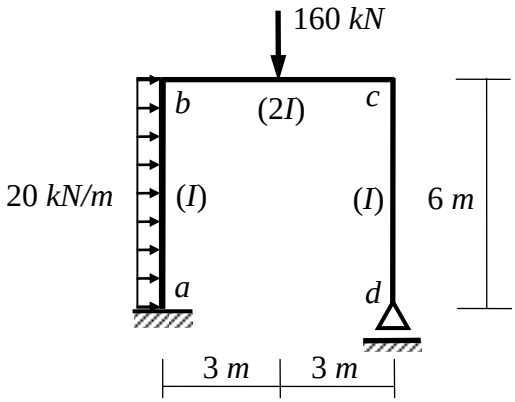
The bending moment and shear force diagram are shown below.



Question (2): (10 Marks)

For the shown frame with variable moment of inertia, using the **Consistent Deformations** (**Virtual Work**) method, draw the bending moment diagram due to the given loads.

E is constant. The relative moments of inertia are given between brackets.



Solution of Question (2)

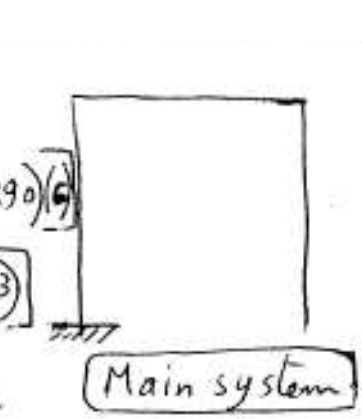
$$\delta_{10} = \int \frac{M_0 M_1}{EI}\,dl$$

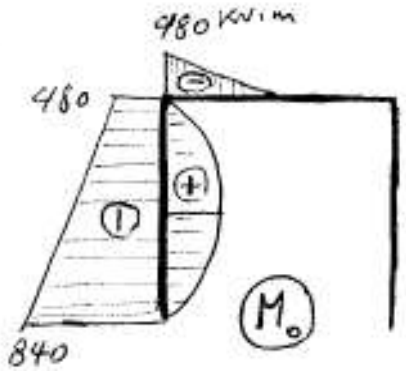
$$= \frac{1}{EI}\left[-\left(\frac{480+840}{2}\times 6\right)(6) + \left(\frac{2}{3}\times 6\times 90\right)(6)\right]$$

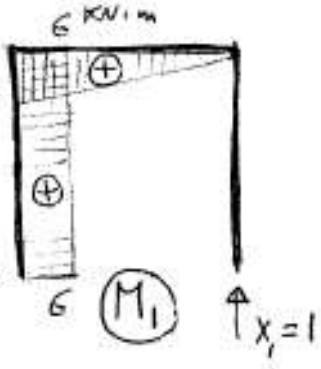
$$+ \frac{1}{2EI}\left[-\left(\frac{1}{2}\times 3\times 480\right)\left(3+\frac{2}{3}\times 3\right)\right]$$

$$= \frac{-2376+2160}{EI} + \frac{-3600}{2EI}$$

$$= \boxed{-\frac{23400}{EI}}$$



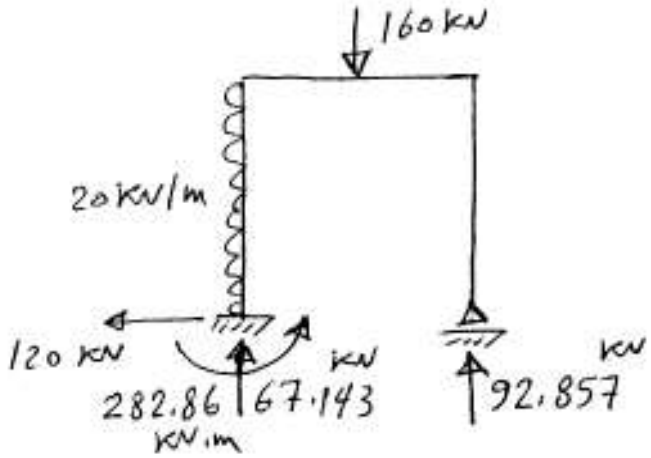


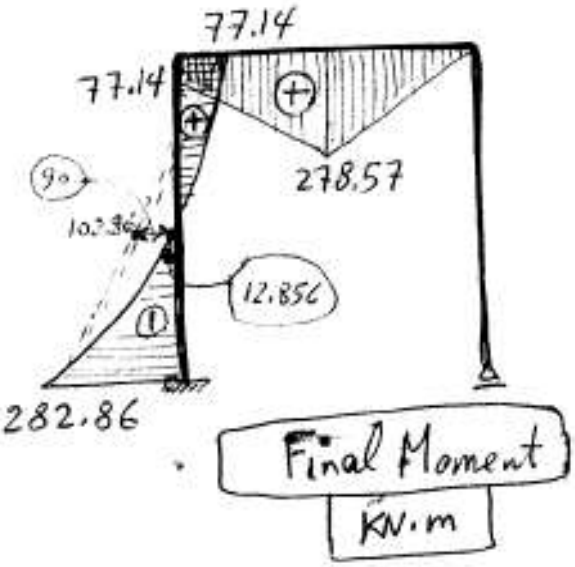


$$\delta_{11} = \int \frac{M_1 M_1}{EI}\,dl = \frac{1}{EI}\left[(6\times 6)(6)\right] + \frac{1}{2EI}\left[\left(\frac{1}{2}\times 6\times 6\right)\left(\frac{2}{3}\times 6\right)\right]$$

$$= \frac{216}{EI} + \frac{72}{2EI} = \boxed{\frac{252}{EI}}$$

$$\delta_{10} + X_1 \delta_{11} = 0 \Longrightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-23400}{252} = \boxed{92.857\ \text{KN}}$$





With my best wishes

Dr. M. Abdel-Kader