

رقم الكود:

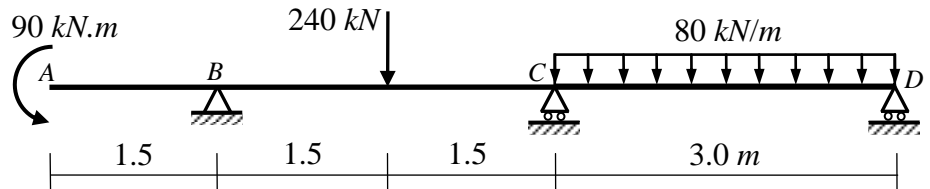
الاسم:

امتحان منتصف الترم

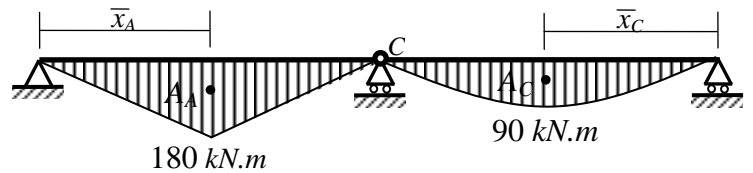
- The Exam consists of **2** questions in **2** pages.

**Question (1): (10 Marks)**

Using the three-moments equation, draw the shear force and bending moment diagrams for the shown beam



**Solution of Question (1)**



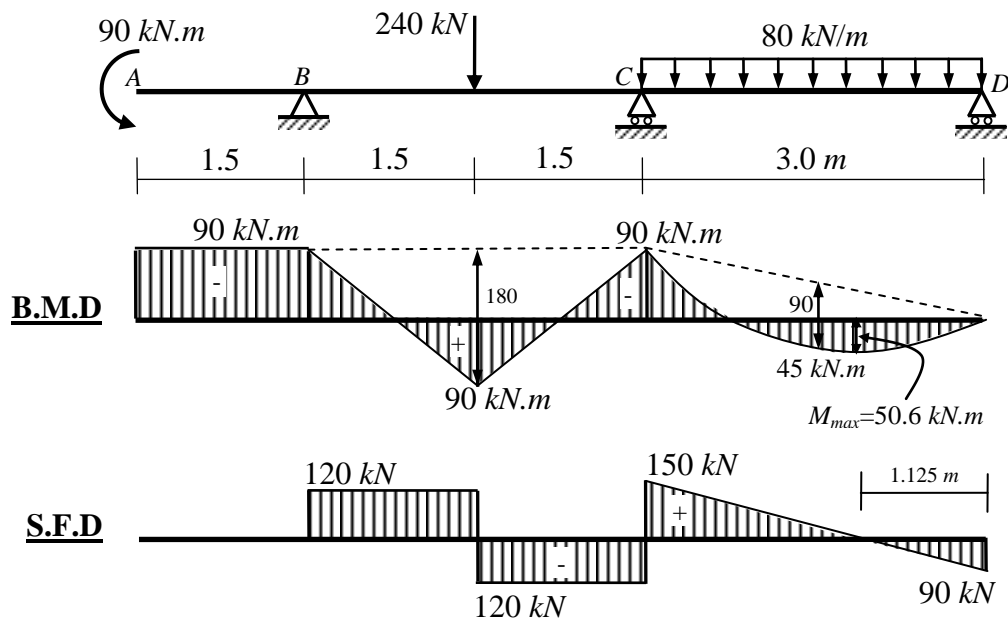
Applying three-moments equation for the spans **BC** and **CD**:

$$M_B(3) + 2M_C(3+3) + M_D(3) = -6 \left( \frac{(0.5 \times 3 \times 180)1.5}{3} + \frac{(2/3 \times 3 \times 90)1.5}{3} \right)$$

$$(-90)(3) + 2M_C(6) + (0)(3) = -1350$$

$$12M_C = -1080 \quad \rightarrow \quad \boxed{M_C = -90 \text{ kN.m}}$$

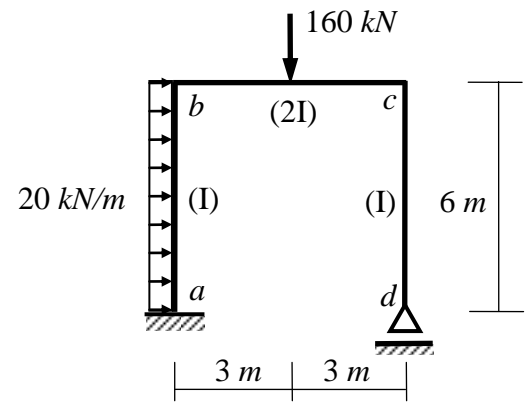
The bending moment and shear force diagrams are shown below.



**Question (2): (10 Marks)**

For the shown frame with variable moment of inertia, using the **Consistent Deformations (Virtual Work)** method, draw the bending moment diagram due to the given loads.

$E$  is constant. The relative moments of inertia are given between brackets.



**Solution of Question (2)**

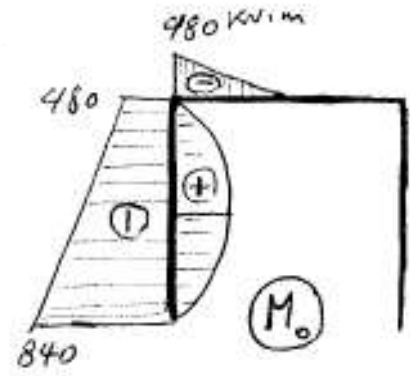
$$\delta_{10} = \int \frac{M_0 M_1}{EI} dl$$

$$= \frac{1}{EI} \left[ \left( \frac{480+840}{2} \times 6 \right) (6) + \left( \frac{2}{3} \times 6 \times 90 \right) (6) \right]$$

$$+ \frac{1}{2EI} \left[ - \left( \frac{1}{2} \times 3 \times 480 \right) \left( 3 + \frac{2}{3} \times 3 \right) \right]$$

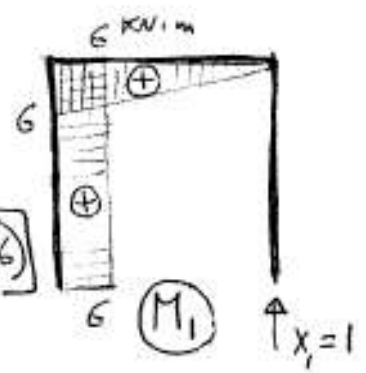
$$= \frac{-2376 + 2160}{EI} + \frac{-3600}{2EI}$$

$$= \boxed{\frac{-23400}{EI}}$$

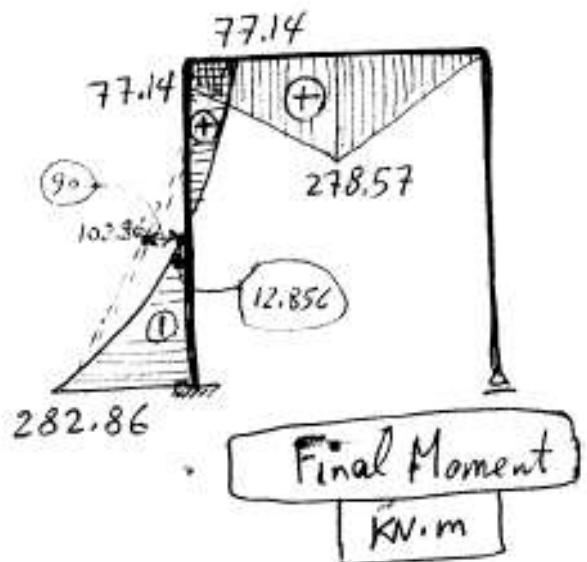
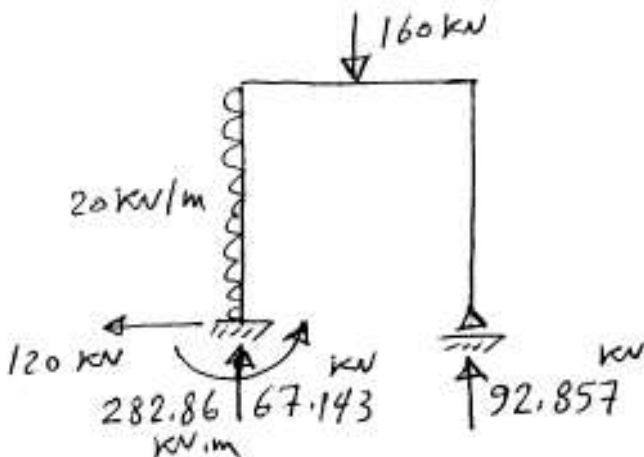


$$\delta_{11} = \int \frac{M_1 M_1}{EI} dl = \frac{1}{EI} [(6 \times 6)(6)] + \frac{1}{2EI} \left[ \left( \frac{1}{2} \times 6 \times 6 \right) \left( \frac{2}{3} \times 6 \right) \right]$$

$$= \frac{216}{EI} + \frac{72}{2EI} = \boxed{\frac{252}{EI}}$$



$$\delta_{10} + X_1 \delta_{11} = 0 \Rightarrow X_1 = - \frac{\delta_{10}}{\delta_{11}} = - \frac{-23400}{252} = \boxed{92.857 \text{ kN}}$$



With my best wishes

Dr. M. Abdel-Kader