Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (4)**
Course Code : **CIV 302**

Academic Year : **2013–2014**
Semester : **Second**
Level : **3rd**
Time : **3 Hours**
Date : **7 / 6 / 2014**
Examiner: ***Dr. M. Abdel-Kader***

Second Semester Final Examination

- Attempt all questions.
- The Exam consists of **4** questions in **1** page.
- Maximum grade is **60 Marks**.

Question (1): (15 Marks)

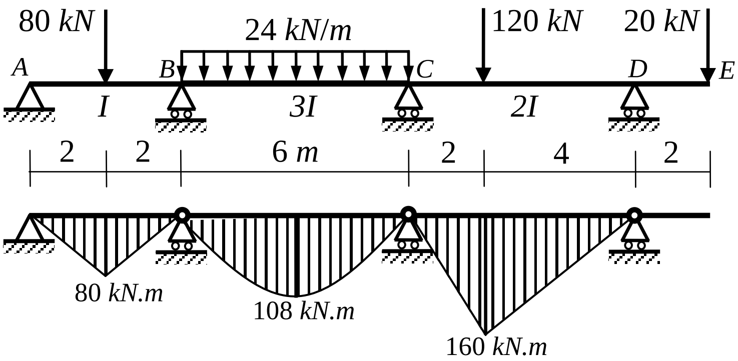
Using the three-moments equation, draw the shear force and bending moment diagrams for the shown continuous beam of variable moment of inertia.

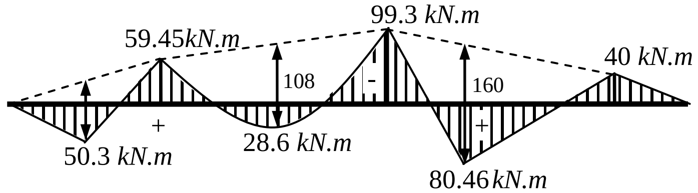
Solution:

The simply supported moment diagram on *AB*, *BC* and *CD* is as shown.

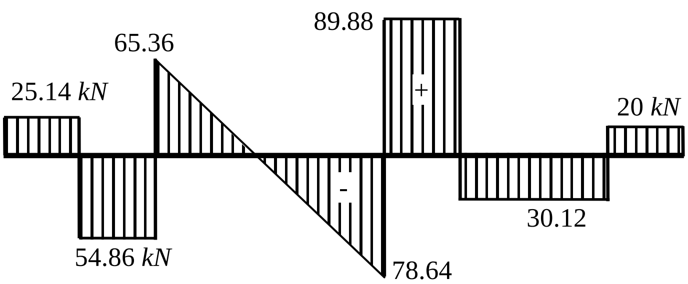
Since the support *A* is simply supported, $M_A = 0$. The moment at *D* is

$$M_D = -20(2) = -40\ kN.m\ .$$





B.M.D



S.F.D

Applying three-moments equation for spans *AB* and *BC*:

$$M_A\left(\frac{4}{I}\right)+2M_B\left(\frac{4}{I}+\frac{6}{3I}\right)+M_C\left(\frac{6}{3I}\right)=-6\left(\frac{(0.5\times4\times80)2}{4I}+\frac{(2/3\times6\times108)3}{6\times3I}\right)$$

$$\therefore\quad 6M_B + M_C = -456 \qquad (1)$$

For spans *BC* and *CD* : (M_D = -40 *kN.m*)

$$M_B\left(\frac{6}{3I}\right)+2M_C\left(\frac{6}{3I}+\frac{6}{2I}\right)+M_D\left(\frac{6}{2I}\right)=-6\left(\frac{(2/3\times6\times108)3}{6\times3I}+\frac{(0.5\times4\times160)2/3\times4+(0.5\times2\times160)(4+2/3)}{6\times2I}\right)$$

or

$$M_B\left(\frac{6}{3I}\right)+2M_C\left(\frac{6}{3I}+\frac{6}{2I}\right)+M_D\left(\frac{6}{2I}\right)=-6\left(\frac{(2/3\times6\times108)3}{6\times3I}+\frac{(0.5\times6\times160)(6+4)/3)}{6\times2I}\right)$$

$$\therefore\quad M_B + 5M_C = -556 \qquad (2)$$

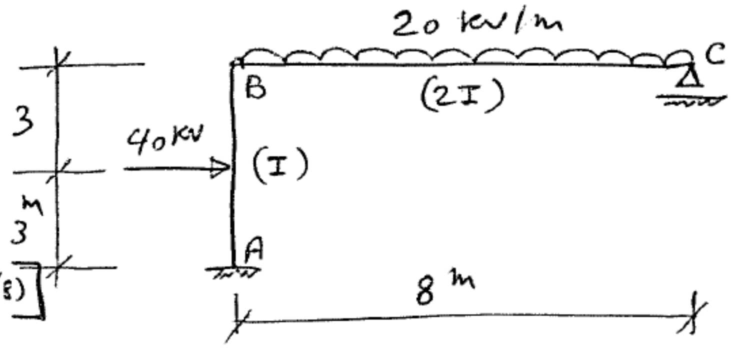
Solving Eqs. (1) and (2) yields M_B = – 59.448 = – 59.45 *kN.m* and M_C = – 99.3 *kN.m*.

R_A = 25.14 *kN*, R_B = 120.22 *kN*, R_C = 168.52 *kN* and R_D = 50.12 *kN*.

The bending moment and shear force diagram are shown above.

Question (2): (15 Marks)

For the shown frame with variable moment of inertia, **using the virtual work method**, find the reactions at the supports *A* and *C*. The relative moments of inertia are given between brackets and *E* is constant.

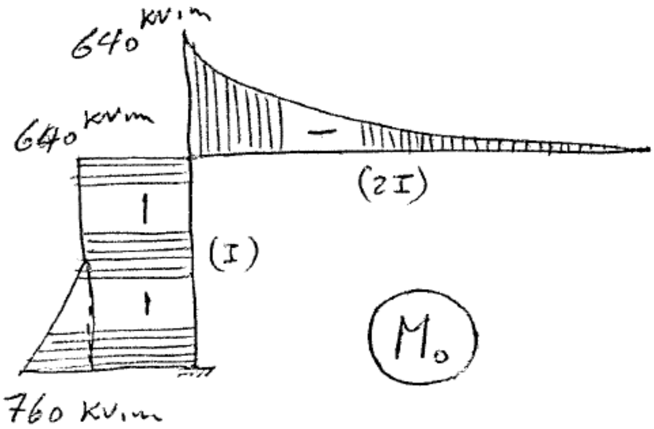


$$\delta_{10} = \int \frac{M_o M_1}{EI} dl$$

$$= \frac{1}{EI}\left[\left(-6\times 640\right)(8) + \left(-\tfrac{1}{2}\times 3\times 120\right)(8)\right] + \frac{1}{2EI}\left[\left(-\tfrac{1}{3}\times 8\times 640\right)\left(\tfrac{3}{4}\times 8\right)\right]$$

$$= \frac{-32160}{EI} - \frac{5120}{EI}$$

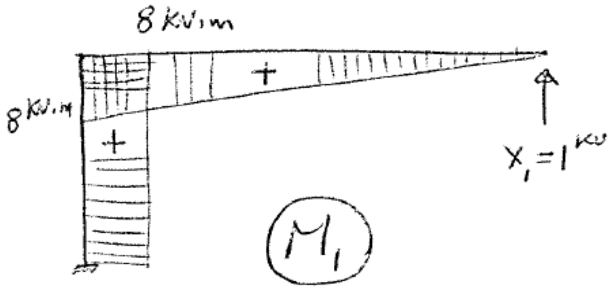
$$\boxed{\delta_{10} = \frac{-37280}{EI}}$$



$$\delta_{11} = \int \frac{M_1 M_1}{EI} dl$$

$$= \frac{1}{EI}\left[(6\times 8)(8)\right] + \frac{1}{2EI}\left[\left(\tfrac{1}{2}\times 8\times 8\right)\left(\tfrac{2}{3}\times 8\right)\right]$$

$$\boxed{\delta_{11} = \frac{469.3333}{EI}}$$



$$\delta_{10} + X_1 \delta_{11} = 0$$

$$\therefore \quad X_1 = -\frac{-37280}{469.3333} = 79.43 \text{ kN}$$

$$\boxed{C_y = X_1 = 79.43 \text{ kN} \uparrow}$$

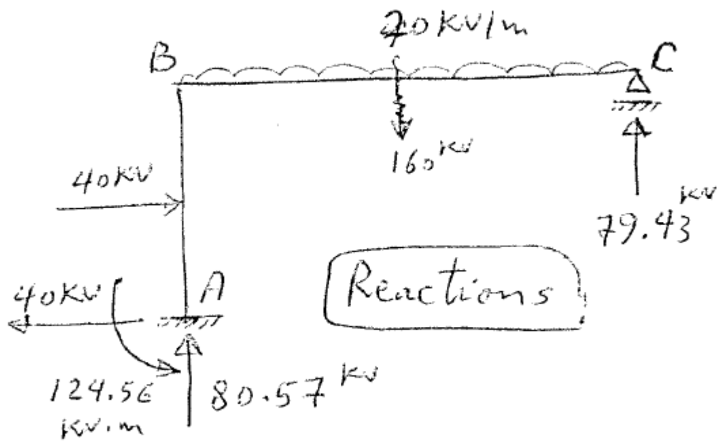
$$\Sigma F_y = 0$$

$$A_y + C_y - 20\times 8 = 0$$

$$\boxed{A_y = 80.57 \text{ kN} \uparrow}$$

$$\Sigma F_x = 0$$

$$\boxed{A_x = 40 \text{ kN} \leftarrow}$$



$$\Sigma M_A = 0$$

$$-M_A - 79.43(8) + (20\times 8)(4) + 40(3) = 0$$

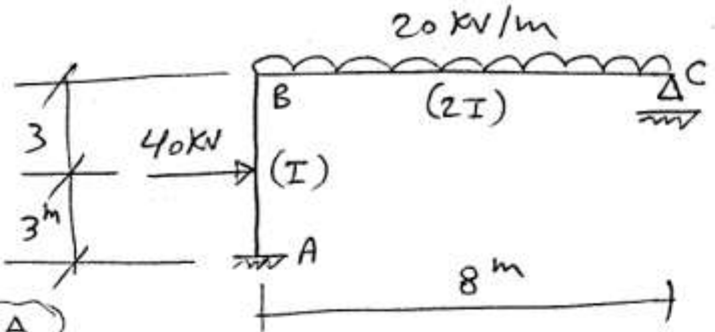
$$\boxed{M_A = 124.56 \text{ kN.m}}$$

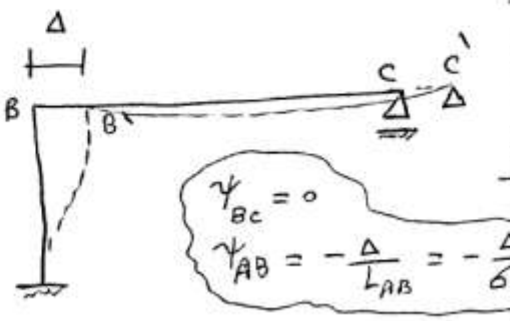
Question (3): **(15 Marks)**

For the frame of Question (2), **using the slope deflection method**,

(a) Find the rotation at *B* (θ_B) and the sway of the frame Δ.

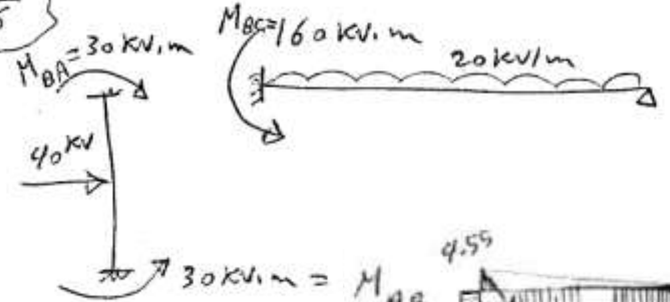
(b) Draw the bending moment diagram.





$\psi_{BC} = 0$

$\psi_{AB} = -\frac{\Delta}{L_{AB}} = -\frac{\Delta}{6}$

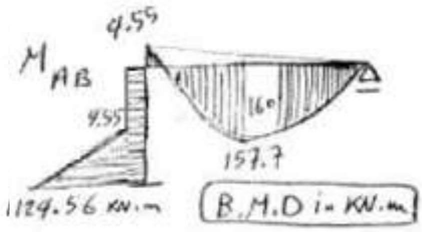


* Unknowns: Θ_B and Δ

* Equilibrium equations

$$\Sigma M_B = M_{BA} + M_{BC} = 0$$

$$\Sigma F_x = 40 + A_x = 0$$



$$M_{BA} = M^F_{BA} + \frac{2EI}{L}(2\theta_B + \theta_A + 3\psi_{AB})$$

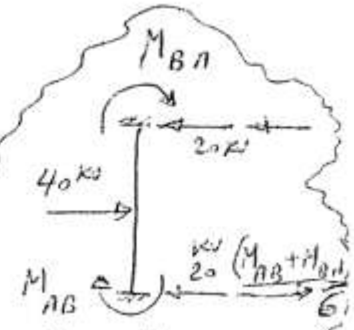
$$= 30 + \frac{2EI}{6}\left(2\theta_B + 0 - 3\frac{\Delta}{6}\right) = 30 + EI\left(\frac{2}{3}\theta_B - \frac{\Delta}{6}\right)$$

$$M_{BC} = M^F_{BC} + \frac{3E(2I)}{L}(\theta_B + \cancel{\psi_{BC}}) = -160 + EI\left(\frac{3}{4}\theta_B\right)$$

$$* \ \Sigma M_B = -130 + EI\left(\frac{17}{12}\theta_B - \frac{\Delta}{6}\right) = 0$$

$$\boxed{\frac{17}{12}\theta_B - \frac{\Delta}{6} = \frac{130}{EI}} \quad (1)$$

$$* \ \Sigma F_x = 40 + \frac{M_{AB} + M_{BA}}{6} - 20 = 0$$



$$A_x = \frac{M_{AB} + M_{BA}}{6} - 20$$

But $B_{AB} = -30 + \frac{2EI}{6}\left(\theta_B - \frac{\Delta}{6}\right) = -30 + EI\left(\theta_B/3 - \frac{\Delta}{6}\right)$

$$\Sigma F_x = 20 + \frac{1}{6}\left[\cancel{-30} + EI\left(\frac{\theta_B}{3} - \frac{\Delta}{6}\right) + \cancel{30} + EI\left(\frac{2}{3}\theta_B - \frac{\Delta}{6}\right)\right]$$

$$= 20 + \frac{EI}{6}\left(\theta_B - \frac{\Delta}{3}\right)$$

$$\boxed{\Theta_B \mp \frac{\Delta}{3} = \frac{\pm 120}{EI}} \quad (2)$$

$$(\times 2) \rightarrow \boxed{\frac{17}{6}\theta_B - \frac{\Delta}{3} = \frac{260}{EI}} \quad (1')$$

$$\rightarrow \frac{11}{6}\theta_B = \frac{380}{EI}$$

$$\Theta_B = \boxed{\frac{2280}{11\ EI}} = \boxed{\frac{207.2727}{EI}}$$

$$\Delta = \boxed{\frac{981.82}{EI}} = \boxed{\frac{10800}{11\ EI}}$$

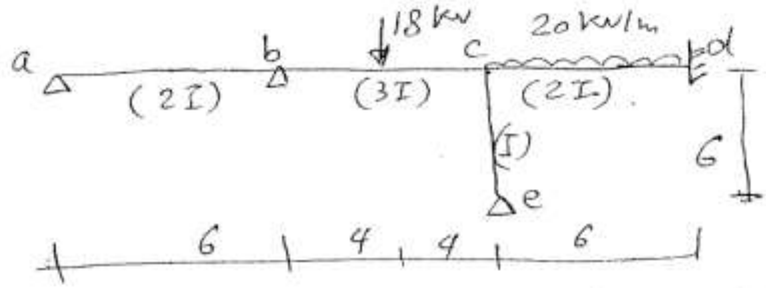
$M_{AB} = 124.545$ kN.m

$M_{BA} = 4.54545$ kN.m

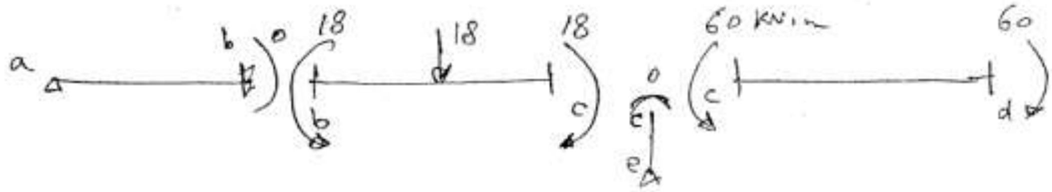
$M_{BC} = 4.54545$ kN.m

Question (4): (15 Marks)

For the shown frame with variable moment of inertia, **using the moment distribution method**, draw the bending moment diagram. *E* is constant. The relative moments of inertia are given between brackets.



F.E.M



D.F.

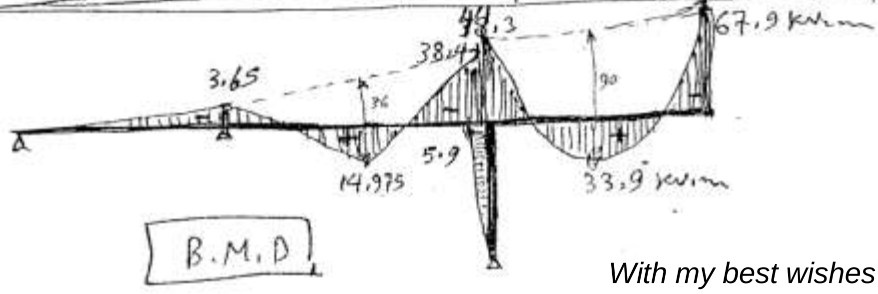
Joint b $\quad K_{ba} = \frac{3E(2I)}{6} = EI \quad K_{bc} = \frac{4E(3I)}{8} = 1.5\,EI$

$\boxed{D.F_{ba} = 0.4}$ $\quad \boxed{D.F_{bc} = 0.6}$

Joint c $\quad K_{cb} = 1.5\,EI \quad K_{ce} = \frac{3EI}{6} = 0.5\,EI \quad K_{cd} = \frac{4E(2I)}{6} = \frac{4}{3}EI$

$\boxed{D.F_{cb} = 0.45}$ $\quad \boxed{D.F_{ce} = 0.15}$ $\quad \boxed{D.F_{cd} = 0.4}$

Joint	b		c			d
Member	ba	bc	cb	ce	cd	dc
D.F	0.4	0.6	0.45	0.15	0.4	1
F.E.M	0	−18	18	0	−60	60
B.M			18.9	6.3	16.8	
C.O.M		9.45				8.4
B.M	3.42	5.13				
C.O.M			2.565			
B.M			−1.15925	−0.38475	−1.026	
C.O.M		−0.5771				−0.513
B.M	0.231	0.3463				
C.O.F			0.173			
B.M			−0.078	−0.026	−0.07	
M	3.65	−3.65	38.4	5.9	−44.3	67.9



$\boxed{B.M.D}$

With my best wishes

Dr. M. Abdel-Kader