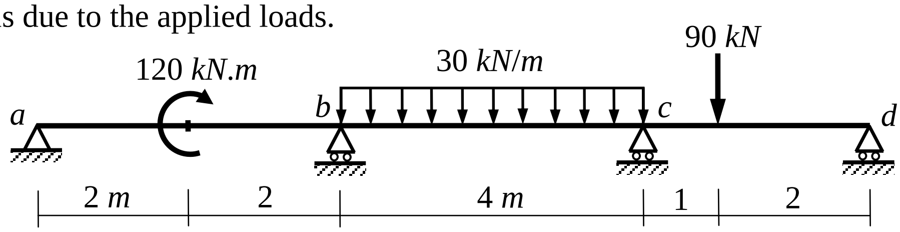
Ministry of Higher Education **Giza Higher Institute for Eng. & Tech.** **Civil Engineering Department**	Academic Year : **2016/2017** Semester : **Second** Level : **3rd**	
Course Name: **Theory of Structures (4)**	Time : **3 Hours**	
Course Code : **CIV 302**	Date : **29 / 5 / 2017**	Examiner: ***Dr. M. Abdel-Kader***
Answer of Final Term Exam		
Total Marks: **60**	No. of Questions: **4**	

Question (1): (15 Marks)

For the shown statically indeterminate continuous beam, **using the three-moment equation**, draw the shear force and bending moment diagrams due to the applied loads.



- The simply supported moment diagram on *abcd* is as shown.

- Applying three-moment equation at *b* (for spans *ba* and *bc*):

$$M_a(L_{ba}) + 2M_b(L_{ba} + L_{bc}) + M_c(L_{bc}) = -6r_b$$

$$(0)(4) + 2M_b(4+4) + M_c(4) = -6(100)$$

$$4M_b + M_c = -150 \quad .... (1)$$

- Applying three-moment equation at *c* (for spans *cb* and *cd*):

$$M_b(L_{cb}) + 2M_c(L_{cb} + L_{cd}) + M_d(L_{cd}) = -6r_c$$

$$M_b(4) + 2M_c(4+3) + (0)(3) = -6(130)$$

$$4M_b + 14M_c = -780 \quad .... (2)$$

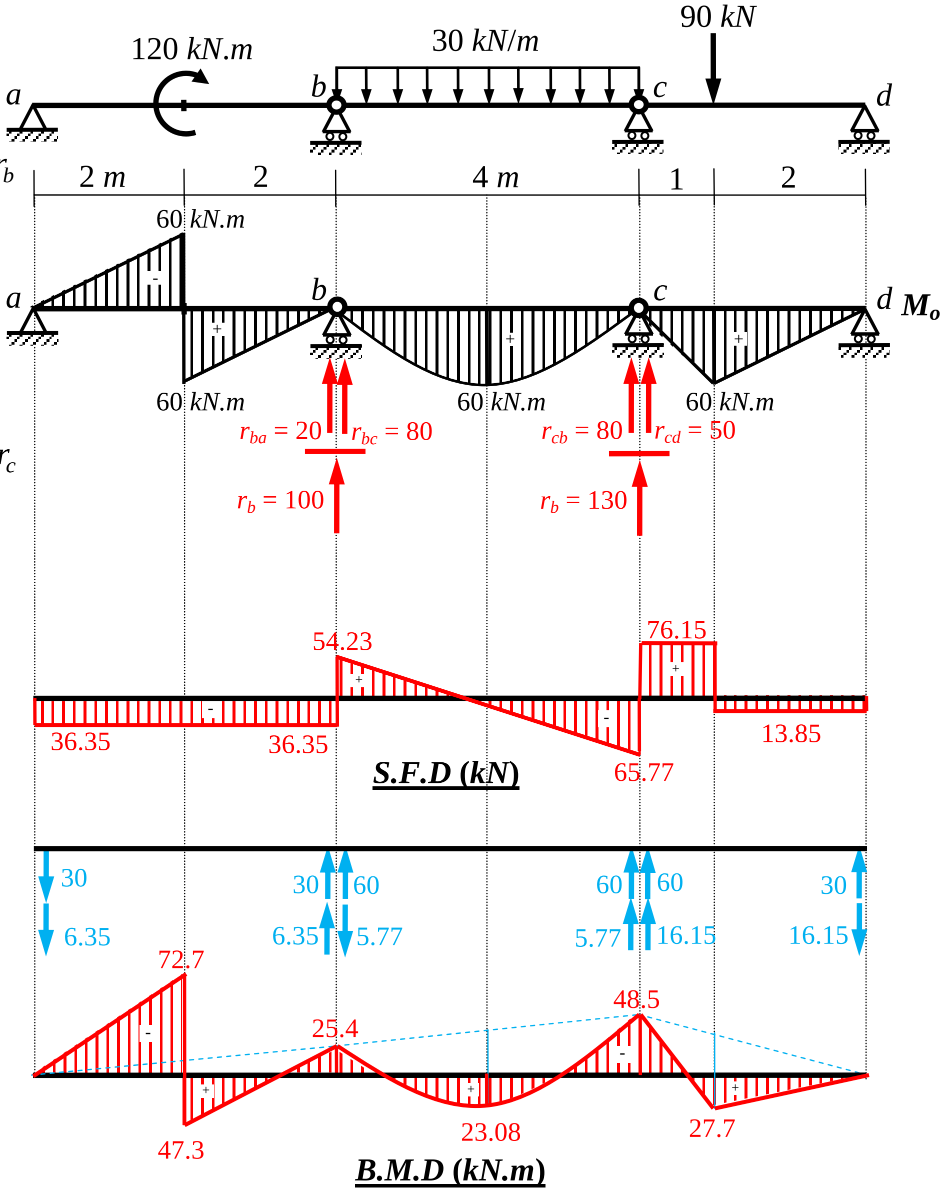
- From Eqs. (1) & (2) →

M_b = - 330/13 = - 25.38 *kN.m*

and M_c = - 630/13 = - 48.46 *kN.m*

The shear force and bending moment diagrams are as shown.

M_{max+ve} in span *bc* = 23.615 *kN.m* at 1.808 *m* from support *b*



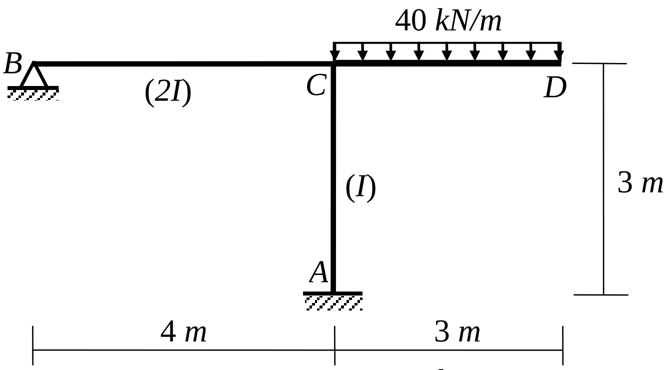
With my best wishes
Dr. M. Abdel-Kader

Question (2): **(15 Marks)**

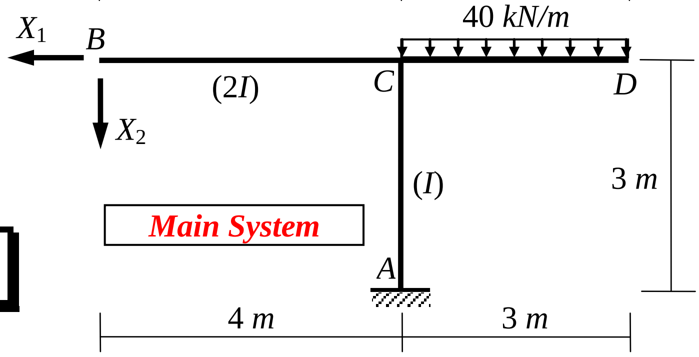
For the shown loaded frame with variable moment of inertia, **using the virtual work method**,
(a) find the reactions at the supports A and B.
(b) draw the bending moment diagram.

The relative moments of inertia are given between brackets and E is constant.

Note:
Take the main system by removing the support B.

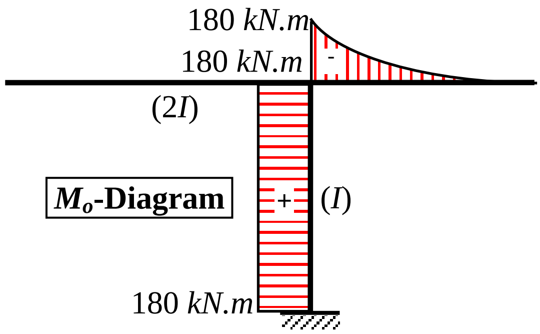


- Main system is as shown.
- Draw M_0-, M_1- and M_2-Diagram.
- Calculations of displacements.



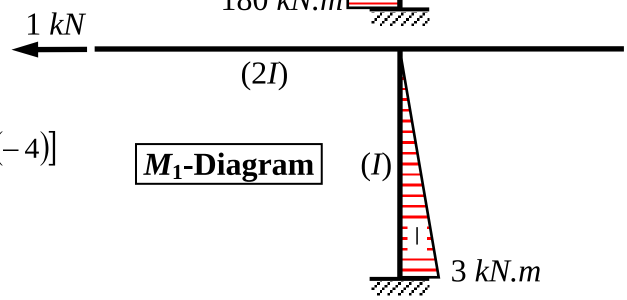
$$\delta_{10} = \int \frac{M_o M_1}{EI} dL = \frac{1}{EI}\left[(3\times180)\left(-\tfrac{1}{2}\times3\right)\right] = \frac{-810}{EI} \qquad \boxed{\delta_{10} = \frac{-810}{EI}}$$

$$\delta_{20} = \int \frac{M_o M_2}{EI} dL = \frac{1}{EI}\left[(3\times180)(-4)\right] = -\frac{2160}{EI} \qquad \boxed{\delta_{20} = \frac{-2160}{EI}}$$



$$\delta_{12} = \delta_{21} = \int \frac{M_1 M_2}{EI} dL = \frac{1}{EI}\left[\left(-\tfrac{1}{2}\times3\times3\right)(-4)\right] = \frac{18}{EI} \qquad \boxed{\delta_{12} = \delta_{21} = \frac{18}{EI}}$$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL = \frac{1}{EI}\left[\left(-\tfrac{1}{2}\times3\times3\right)\left(-\tfrac{2}{3}\times3\right)\right] = \frac{9}{EI} \qquad \boxed{\delta_{11} = \frac{9}{EI}}$$



$$\delta_{22} = \int \frac{M_2 M_2}{EI} dL = \frac{1}{2EI}\left[\left(-\tfrac{1}{2}\times4\times4\right)\left(-\tfrac{2}{3}\times4\right)\right] + \frac{1}{EI}\left[(-3\times4)(-4)\right]$$

$$= \frac{58.667}{EI} \qquad \boxed{\delta_{22} = \frac{58.667}{EI}}$$

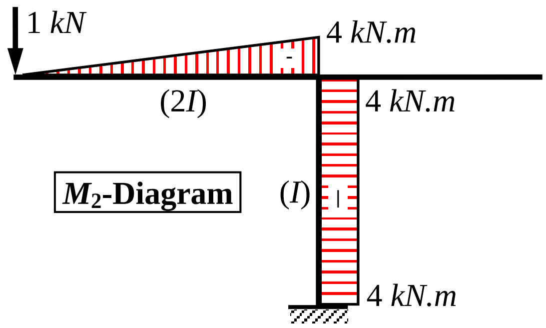
$$\delta_{10} + X_1\delta_{11} + +X_2\delta_{12} = 0$$

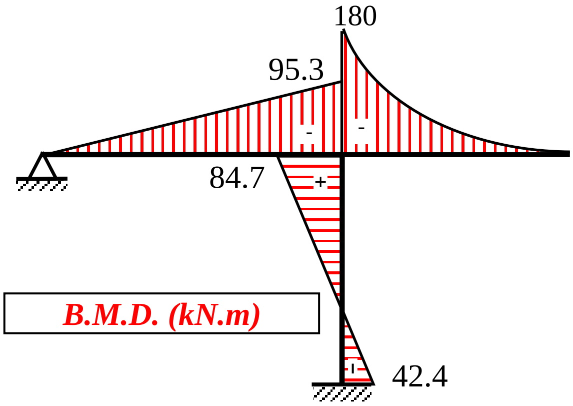
$$\delta_{20} + X_1\delta_{21} + +X_2\delta_{22} = 0$$

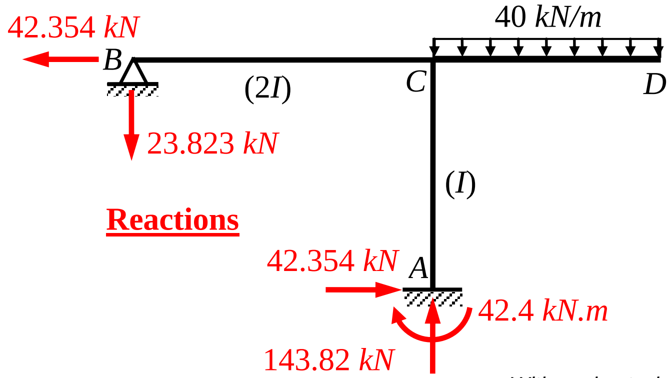
$-810 + 9X_1 + \quad 18X_2 = 0 \quad \rightarrow \times\text{-}2 \quad 1620 - 18X_1 - \quad 36X_2 = 0$

$-2160 + 18X_1 + 58.667X_2 = 0 \qquad -2160 + 18X_1 + 58.667X_2 = 0$

$\therefore$ $\boxed{X_1 = +42.354\ kN}$ *and* $\boxed{X_2 = +23.823\ kN}$



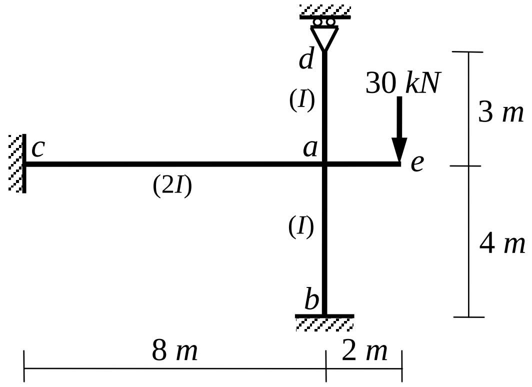




With my best wishes
Dr. M. Abdel-Kader

Question (3): (15 Marks)

For the shown loaded frame, **using the slope deflection method**, draw the bending moment diagram. Note that E is constant and the relative moments of inertia are given between brackets.



- Unknown displacements: only θ_a.

- The static equilibrium equation required to determine this unknown is

$$\sum M_a = M_{ab} + M_{ac} + M_{ad} + M_{ae} = 0$$

Note that M_{ad} and M_{ae} are determined: $M_{ad} = 0$ and $M_{ae} = -60\ kN.m$

- Fixed end moments:

$$M^F_{ab} = M^F_{ab} = 0 \quad \text{and} \quad M^F_{ac} = M^F_{ca} = 0$$

- The slope deflection equations are:

$$M_{ab} = M^F_{ab} + \frac{2EI}{L}(2\theta_a + \theta_b + 3\psi_{ab}) = 0 + \frac{2E(I)}{4}(2\theta_a + 0 - 0) = EI\theta_a$$

$$M_{ac} = M^F_{ac} + \frac{2EI}{L}(2\theta_a + \theta_c + 3\psi_{ac}) = 0 + \frac{2E(2I)}{8}(2\theta_a + 0 - 0) = EI\theta_a$$

$$M_{ad} = 0 \quad \text{and} \quad M_{ae} = \text{-}120\ kN.m$$

- Substituting into the static equilibrium equation,

$$\sum M_a = M_{ab} + M_{ac} + M_{ad} + M_{ae} = EI\theta_a + EI\theta_a + 0 + (\text{-}120) = 0$$

$$2EI\theta_a = 60 \qquad \boxed{\theta_a = 30/EI}$$

- Back-substituting by θ_a into the slope deflection equations, the end moments become:

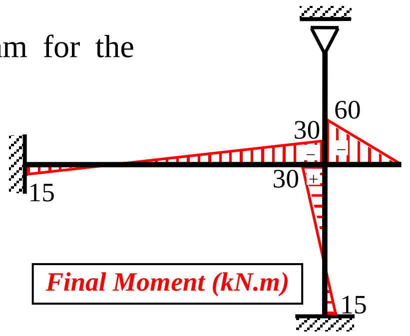
$$M_{ab} = EI\theta_a = EI(60/EI) = 30\ kN.m \qquad M_{ba} = 0.5EI\theta_a = 0.5EI(60/EI) = 15\ kN.m$$

$$M_{ac} = EI\theta_a = EI(60/EI) = 30\ kN.m \qquad M_{ca} = 0.5EI\theta_a = 0.5EI(60/EI) = 15\ kN.m$$

$$M_{ad} = 0$$

$$M_{ae} = \text{-}60\ kN.m$$

- The final bending moment diagram for the whole frame is as shown.



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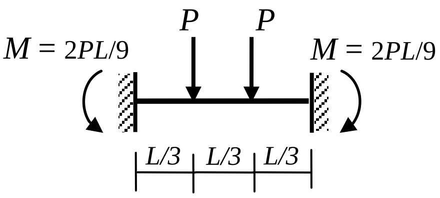
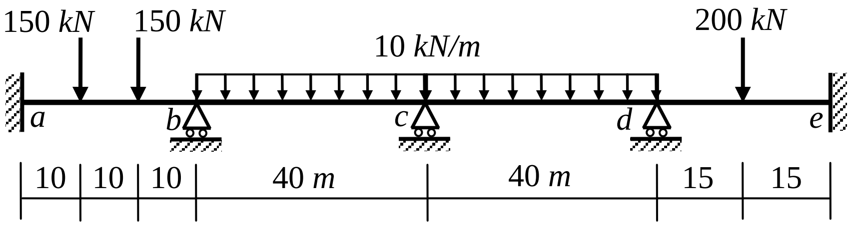
Question (4): (15 Marks)

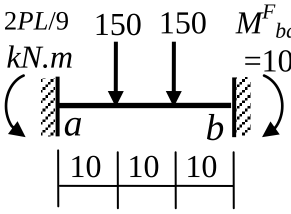
Using **the moment distribution method,** draw the bending moment diagram for the shown statically indeterminate continuous beam. Assume that ***EI*** is constant.



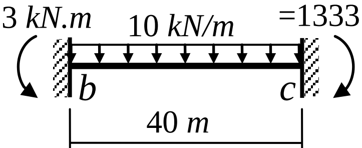


- Fixed end moments:

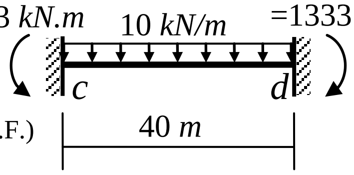
$M^F_{ab} = 2PL/9$ $=1000\ kN.m$ $\qquad M^F_{ba} = 2PL/9$ $=1000\ kN.m$



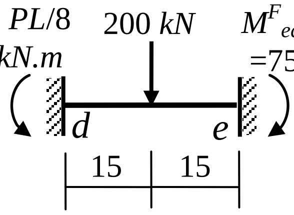
$M^F_{bc} = wL^2/12$ $=1333.3\ kN.m$ $\qquad M^F_{cb} = wL^2/12$ $=1333.3\ kN.m$



$M^F_{cd} = wL^2/12$ $=1333.3\ kN.m$ $\qquad M^F_{dc} = wL^2/12$ $=1333.3\ kN.m$



$M^F_{de} = PL/8$ $=750\ kN.m$ $\qquad M^F_{ed} = PL/8$ $=750\ kN.m$



- Distribution factors (D.F.)

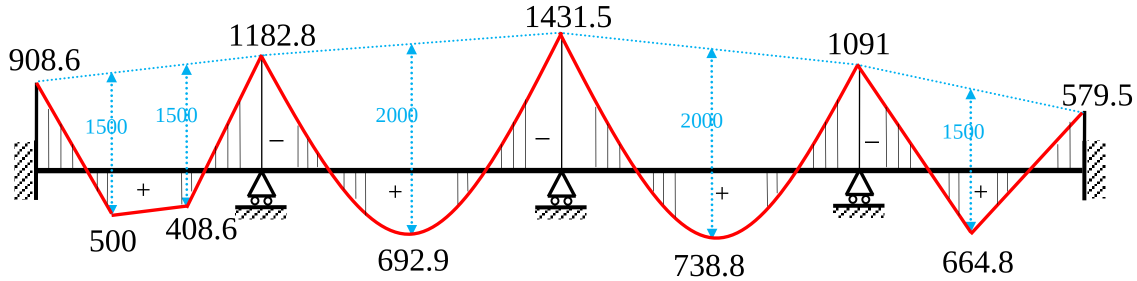
Joint *b*: $k_{ba} = \dfrac{4EI}{L_{ba}} = \dfrac{4}{30}EI$ and $k_{bc} = \dfrac{4EI}{L_{bc}} = \dfrac{4}{40}EI = \dfrac{3}{30}EI$ then $\sum k_i = k_{ba} + k_{bc} = \dfrac{7}{30}EI$

$D.F._{ba} = \dfrac{k_{ba}}{\sum k_i} = \dfrac{4}{7}$ & $D.F._{bc} = \dfrac{k_{bc}}{\sum k_i} = \dfrac{3}{7}$ $\qquad \boxed{D.F._{ba} = 4/7}$ $\boxed{D.F._{bc} = 3/7}$

Joint *c*: → $\boxed{D.F._{cb} = 0.5}$ $\boxed{D.F._{cd} = 0.5}$

Joint *d*: $k_{dc} = \dfrac{4EI}{L_{dc}} = \dfrac{4}{40}EI = \dfrac{3}{30}EI$ and $k_{de} = \dfrac{4EI}{L_{de}} = \dfrac{4}{30}EI$ then $\sum k_i = k_{dc} + k_{de} = \dfrac{7}{30}EI$ $\qquad \boxed{D.F._{dc} = 3/7}$ $\boxed{D.F._{de} = 4/7}$

Joint	a	b		c		d		e
Member	ab	ba	bc	cb	cd	dc	de	ed
D.F.	1	4/7	3/7	0.5	0.5	3/7	4/7	1
F.E.M.	-1000	1000	-1333.3	1333.3	-1333.3	1333.3	-750	750
B.M.		190.46	142.84			-250	-333.3	
C.O.M.	95.23			71.42	-125			-166.65
B.M.				26.79	26.79			
C.O.M.			13.4			13.4		
B.M.		-7.66	-5.74			-5.74	-7.66	
C.O.M.	-3.83			-2.87	-2.87			-3.83
B.M.				2.87	2.87			
M_{final}	-908.6	1182.8	-1182.8	1431.5	-1431.5	1091	-1091	579.5



Bending Moment Diagram (kN.m)

With my best wishes
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