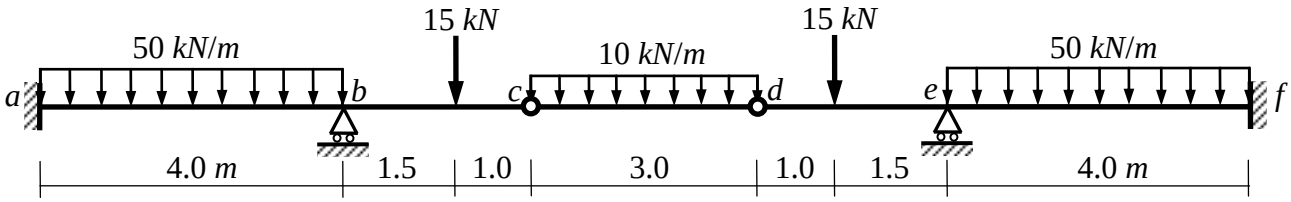
Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (4)**
Course Code : **CIV 302**

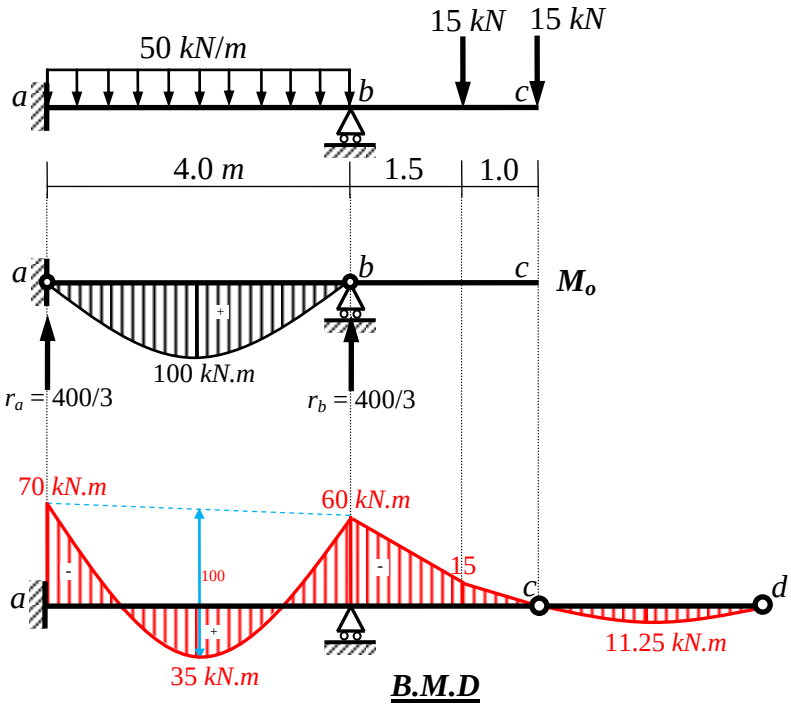
Academic Year : **2015–2016**
Semester : **Second**
Level : **3rd**
Time : **3 Hours**
Date : **4 / 6 / 2016**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Second Semester Final Exam

Question (1): (12 Marks)



(a) Due to symmetry, only part *abc* may be analyzed. The simply supported moment diagram on *ab* is as shown.



B.M.D

The moment at *b*,

M_b = -15(2.5)-15(1.5) = -60 *kN.m*

Applying three-moment equation at *a* (for spans $a_o a$ and *ab*):

$$2M_a(L_{ab}) + M_b(L_{ab}) = -6r_C$$

$$2M_a(4) + (-60)(4) = -6(400/3) = -800$$

$$\rightarrow \boxed{M_a = -70\ kN.m}$$

The bending moment diagram is as shown.

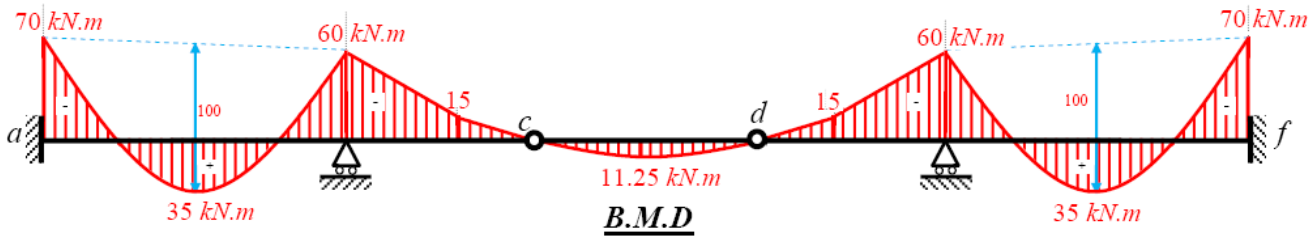
(b) Applying the three-moment equation at *a*: (note that $\Delta_a = 0$, $\Delta_b = 10\ mm = 0.01\ m$ and $M_b = 0$)

$$2M_a(4) + (0)(4) = 6EI\left(\frac{(\Delta_a - \Delta_{ao})}{4} + \frac{(\Delta_a - \Delta_b)}{4}\right) = 6EI\left(\frac{(0-0)}{4} + \frac{(0-0.01)}{4}\right)$$

$$8M_a = -0.015EI$$

$$M_a = -\frac{0.015 \times 37333}{8} = -70\,kN.m$$

$\therefore$ Percentage increase in M_a = (70/70)×100 = 100 %



B.M.D

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Question (2): (12 Marks)

$$\delta_{10} = \int \frac{M_o M_1}{EI} dL$$

$$= \frac{1}{EI}\left[\left(\frac{1}{3}\times 6\times 900\right)\left(-\frac{3}{4}\times 1\right)\right]$$

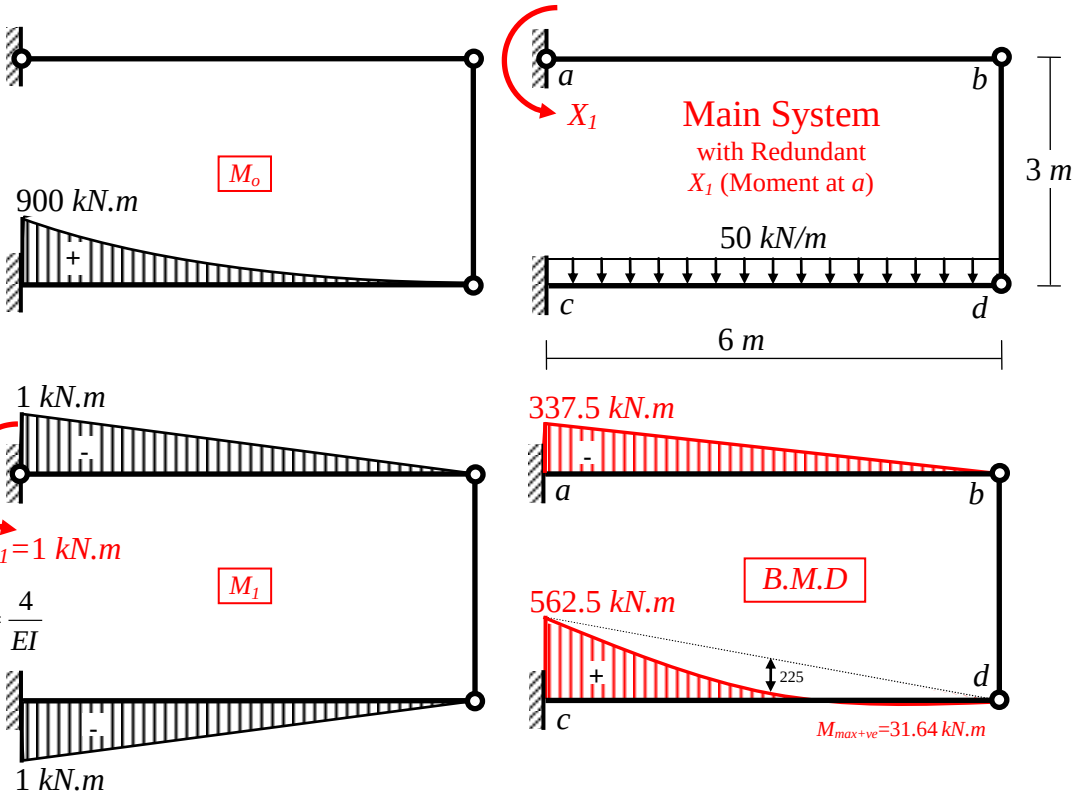
$$= \frac{-1350}{EI}$$

$\delta_{10} = -1350/EI$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$= \frac{2}{EI}\left[\left(-\frac{1}{2}\times 6\times 1\right)\left(-\frac{2}{3}\times 1\right)\right] = \frac{4}{EI}$$

$\delta_{11} = 4/EI$



$$\delta_{10} + X_1\delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-1350}{4} = 337.5 \ kN.m$$

$X_1 = M_a = 337.5 \ kN.m \circlearrowleft$

$M_a = M_{ao} + X_1 \quad M_{a1} = 0 + (337.5)(-1) = -337.5 \ kN.m$

$M_c = M_{co} + X_1 \quad M_{c1} = 900 + (337.5)(-1) = 562.5 \ kN.m$

Question (2):
Another Solution:

$$\delta_{10} = \int \frac{M_o M_1}{EI} dL$$

$$= \frac{1}{EI}\left[\begin{array}{l}\left(-\frac{2}{3}\times 6\times 225\right)\left(\frac{1}{2}\times 1\right) \\ +\left(-\frac{1}{2}\times 6\times 900\right)\left(\frac{2}{3}\times 1\right)\end{array}\right]$$

$$= \frac{-2250}{EI}$$

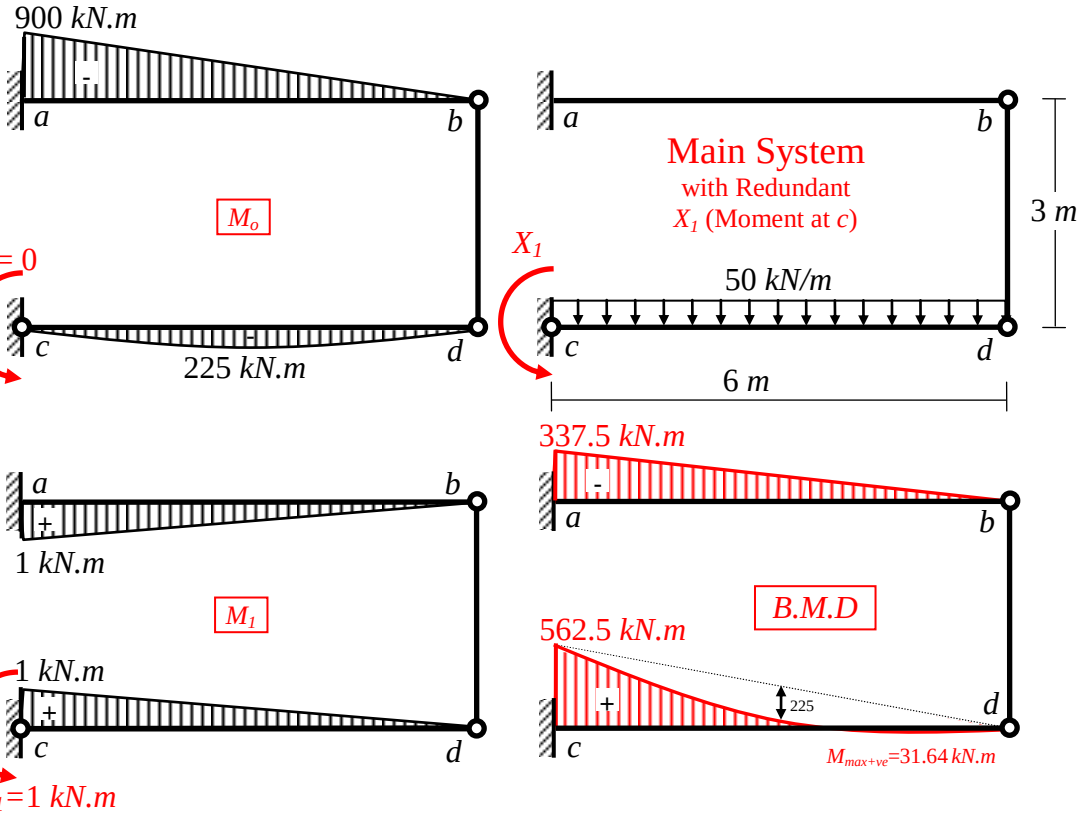
$\delta_{10} = -2250/EI$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$= \frac{2}{EI}\left[\left(\frac{1}{2}\times 6\times 1\right)\left(\frac{2}{3}\times 1\right)\right]$$

$$= \frac{4}{EI}$$

$\delta_{11} = 4/EI$



$$\delta_{10} + X_1\delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-2250}{4} = 562.5 \ kN.m$$

$X_1 = M_c = 562.5 \ kN.m \circlearrowleft$

$M_a = M_{ao} + X_1 \quad M_{a1} = -900 + (562.5)(1) = -337.5 \ kN.m$

$M_c = M_{co} + X_1 \quad M_{c1} = 0 + (562.5)(1) = 562.5 \ kN.m$

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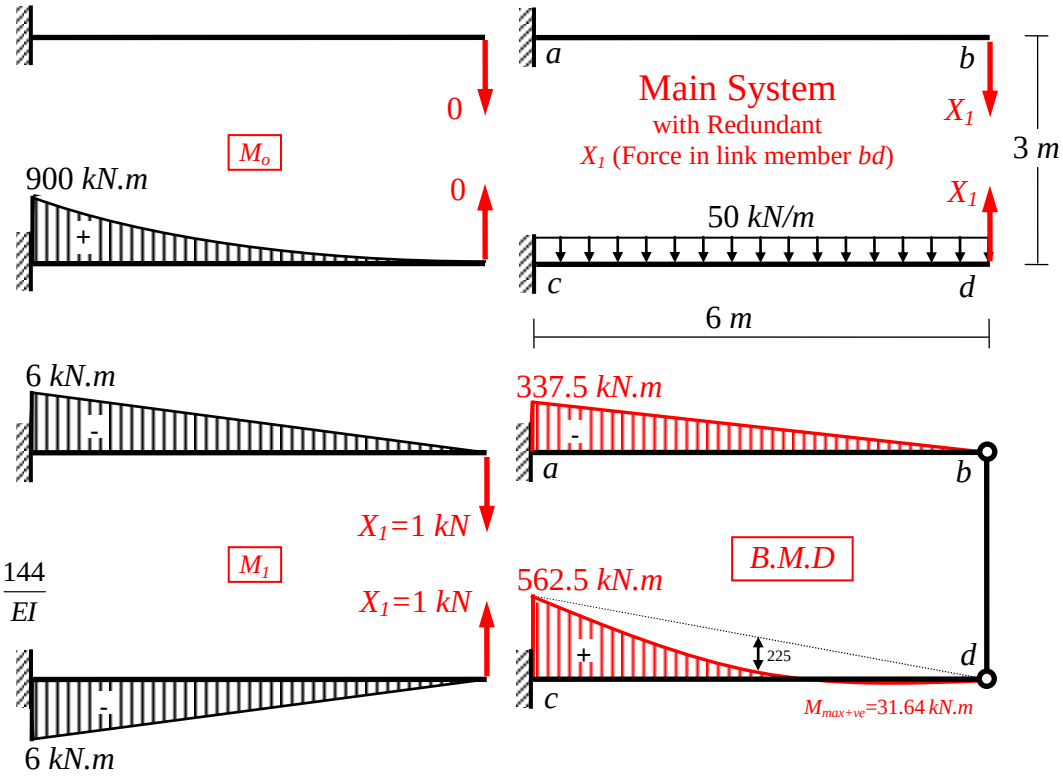
Question (2):
Another Solution:



$$\delta_{10} = \int \frac{M_o M_1}{EI} dL$$

$$= \frac{1}{EI}\left[\left(\frac{1}{3}\times 6\times 900\right)\left(-\frac{3}{4}\times 6\right)\right]$$

$$= \frac{-8100}{EI}$$

$$\delta_{10} = -8100/EI$$

$$\delta_{11} = \int \frac{M_1 M_1}{EI} dL$$

$$= \frac{2}{EI}\left[\left(-\frac{1}{2}\times 6\times 6\right)\left(-\frac{2}{3}\times 6\right)\right] = \frac{144}{EI}$$

$$\delta_{11} = 144/EI$$

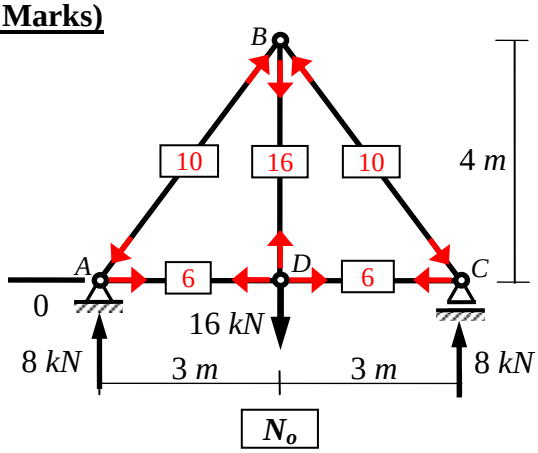
$$\delta_{10} + X_1\delta_{11} = 0 \rightarrow X_1 = -\frac{\delta_{10}}{\delta_{11}} = -\frac{-8100}{144} = 56.25\ kN$$

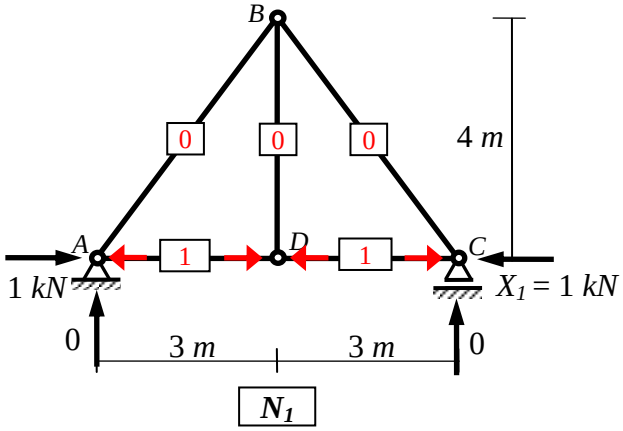
$$X_1 = F_{bd} = 56.25\ kN$$

$M_a = M_{ao} + X_1\ M_{a1} =$ 0 + (56.25)(-6) = -337.5 *kN.m*

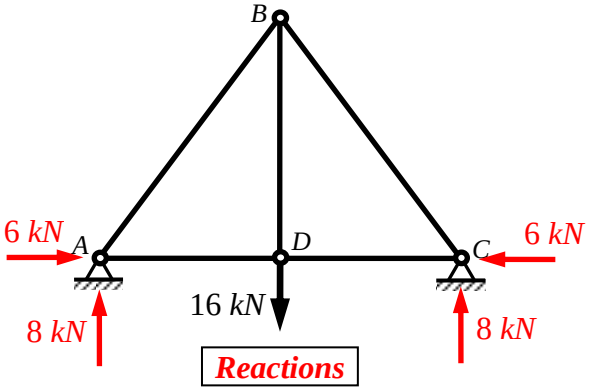
$M_c = M_{co} + X_1\ M_{c1}$ = 900 + (56.25)(-6) = 562.5 *kN.m*

Question (3): (12 Marks)





Member	N_o (kN)	N_1 (kN)	L (m)	EA (kN)	$N_o N_1 L/EA$	$N_1 N_1 L/EA$
AB	-10	0	5	1	0	0
BC	-10	0	5	1	0	0
CD	6	-1	3	1	-18	3
AD	6	-1	3	1	-18	3
BD	16	0	4	1	0	0
Σ					δ_{10} = -36	δ_{11} = 6



$$\delta_{10} = \sum_{i=1}^{i=m} \frac{N_{o\,i} N_{1_i} L_i}{E_i A_i} = -36 \quad \text{and} \quad \delta_{11} = \sum_{i=1}^{i=m} \frac{N_{1\,i} N_{1_i} L_i}{E_i A_i} = 3$$

$$\delta_{Ch} = \delta_{10} + X_1\delta_{11} = 0$$

X_1 (X_1= C_x) can be obtained from the above equation as follows:

$$X_1 = -\frac{\delta_{10}}{\delta_{11}} \rightarrow X_1 = -\frac{-36}{6} = +6\ kN$$

$$X_1 = C_x = 6\ kN \leftarrow$$

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Question (4): (12 Marks)

- Unknown displacements: only θ_a.
- The static equilibrium equation required to determine this unknown is

  $\sum M_a = M_{ab} + M_{ac} + M_{ad} + M_{ae} = 0$

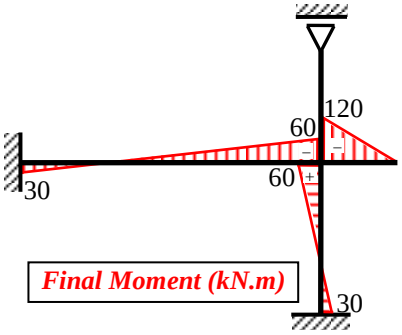
  Note that M_{ad} and M_{ae} are determined; $M_{ad} = 0$ and $M_{ae} = -120\ kN.m$
- Fixed end moments:

  $M^F_{ab} = M^F_{ab} = 0$ and $M^F_{ac} = M^F_{ca} = 0$
- The slope deflection equations are:

$$M_{ab} = M^F_{ab} + \frac{2EI}{L}\left(2\theta_a + \theta_b + 3\psi_{ab}\right) = 0 + \frac{2E(I)}{4}\left(2\theta_a + 0 - 0\right) = EI\theta_a$$

$$M_{ac} = M^F_{ac} + \frac{2EI}{L}\left(2\theta_a + \theta_c + 3\psi_{ac}\right) = 0 + \frac{2E(2I)}{8}\left(2\theta_a + 0 - 0\right) = EI\theta_a$$

$$M_{ad} = 0 \text{ and } M_{ae} = -120\ kN.m$$



- Substituting into the static equilibrium equation,

$$\sum M_a = M_{ab} + M_{ac} + M_{ad} + M_{ae} = EI\theta_a + EI\theta_a + 0 + (-120) = 0$$

$$2EI\theta_a = 120 \qquad \theta_a = 60/EI$$

- Back-substituting by θ_a into the slope deflection equations, the end moments become:

$M_{ab} = EI\theta_a = EI(60/EI) = 60\ kN.m$ $\qquad$ $M_{ba} = 0.5EI\theta_a = 0.5EI(60/EI) = 30\ kN.m$

$M_{ac} = EI\theta_a = EI(60/EI) = 60\ kN.m$ $\qquad$ $M_{ca} = 0.5EI\theta_a = 0.5EI(60/EI) = 30\ kN.m$

$M_{ad} = 0$

$M_{ae} = -120\ kN.m$

- The final bending moment diagram for the whole frame is as shown.

Question (5): (12 Marks)

. - Fixed end moments:

- Distribution factors

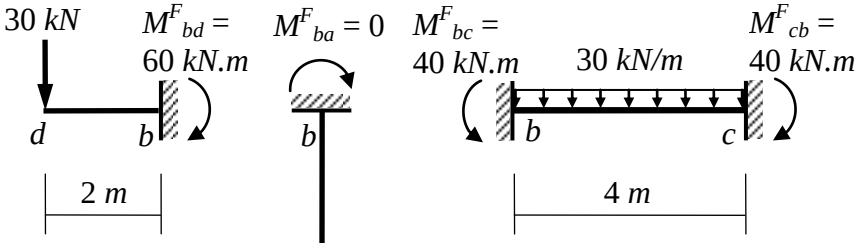
Joint *b*

$$k_{ba} = \frac{4EI}{L_{ba}} = \frac{4EI}{4} = EI \quad ,$$

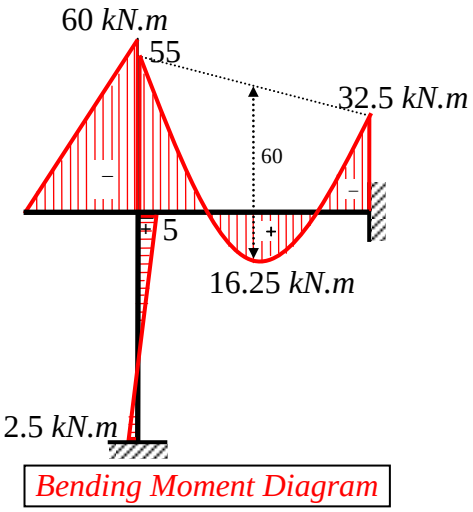
$$k_{bd} = 0 \text{ and } k_{bc} = \frac{4EI}{L_{bc}} = \frac{4E(3I)}{4} = 3EI$$

then $\sum k_i = k_{ba} + k_{bc} + k_{bd} = 4EI$

$D.F._{ba} = 0.25 \quad D.F._{bc} = 0.75 \quad D.F._{bd} = 0$



Joint	*a*	*b*			*c*
Member	*ab*	*ba*	*bd*	*bc*	*cb*
D.F.	1	0.25	0	0.75	1
F.E.M.	0	0	+60	-40	+40
B.M.		-5	0	-15	
C.O.M.	-2.5				-7.5
B.M.					
M_{final}	-2.5	-5	+60	-55	+32.5



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