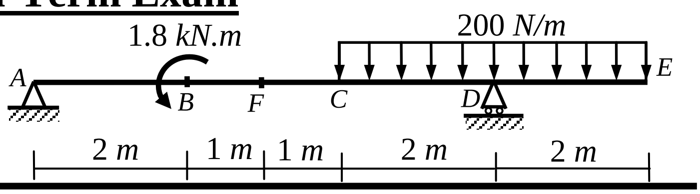
Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (3)**
Course Code : **CIV 301**

Academic Year : **2016–2017**
Semester : **First**
Level : **3rd**
Time : **1½ Hours**
Date : **20 / 11 / 2016**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Mid-Term Exam

Question (1): (10 Marks)

Using the **double integration method**, determine the deflections at ***E*** and ***F*** for the beam loaded as shown, and sketch the elastic curve of the beam. $EI = 100000\ N.m^2$



Solution:

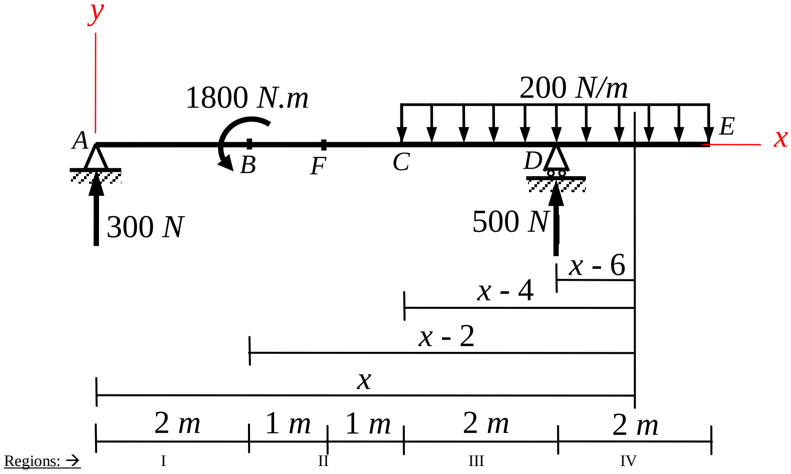
Reactions:

$A_y(6) - 1800 - 200\times4(0) = 0$

$\rightarrow A_y = 300\ N$

$D_y + 300 - 200\times4 = 0$

$\rightarrow D_y = 500\ N$



$M = 300x \,|_{I} - 1800(x-2)^0 \,|_{II} - 200(x-4)^2/2 \,|_{III} + 500(x-6) \,|_{IV}$

$\;\;\;= 300x \,|_{I} - 1800(x-2)^0 \,|_{II} - 100(x-4)^2 \,|_{III} + 500(x-6) \,|_{IV}$

$EI\,y'' = 300x \,|_{I} - 1800(x-2)^0 \,|_{II} - 100(x-4)^2 \,|_{III} + 500(x-6) \,|_{IV}$

$EI\,y' = 150x^2 \,|_{I} - 1800(x-2) \,|_{II} - 100(x-4)^3/3 \,|_{III} + 250(x-6)^2 \,|_{IV} + C_1$

$EI\,y = 50x^3 \,|_{I} - 900(x-2)^2 \,|_{II} - 25(x-4)^4/3 \,|_{III} + 250(x-6)^3/3 \,|_{IV} + C_1 x + C_2$

Boundary Conditions:

At $x = 0$, $y = 0 \rightarrow C_2 = 0$

At $x = 6\ m$, $y = 0 \rightarrow 0 = 50(6^3) - 900(4^2) - 25(2)^4/3 + 6C_1 \rightarrow C_1 = 5600/9\ N{\cdot}m^3$

So, the general equation of the deflection y at any distance x is,

$EI\,y = 50x^3 - 900(x-2)^2 - 25(x-4)^4/3 + 250(x-6)^3/3 + 5600x/9$

The deflection at *E* (δ_F), at *x* = 8 *m* **(in the last region)**

$EI\,\delta_E = 50(8)^3 \,|_{I} - 900(8-2)^2 \,|_{II} - 25(8-4)^4/3 \,|_{III} + 250(8-6)^3/3 \,|_{IV} + 5600(8)/9$

$EI\,\delta_E = 50(8)^3 - 900(6)^2 - 25(4)^4/3 + 250(2)^3/3 + 5600(8)/9 = -29600/9$

$\delta_E = -29600/(9\times100000) = -0.0328889\ m$

$\boxed{\delta_E = 32.9\ mm \downarrow}$

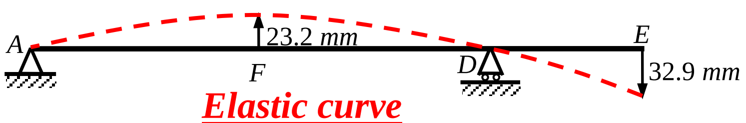
The deflection at *F* (δ_F), at *x* = 3 *m* **(in region II)**

$EI\,\delta_F = 50(3)^3 \,|_{I} - 900(3-2)^2 \,|_{II} - ~~25(3-4)^4/3~~ \,|_{III} + ~~250(3-6)^3/3~~ \,|_{IV} + 5600(3)/9$

$EI\,\delta_F = 50(3)^3 - 900(1)^2 + 5600(3)/9 = 6950/3$

$\delta_F = 6950/(3\times100000) = 0.02317\ m$

$\boxed{\delta_F = 23.2\ mm \uparrow}$



Elastic curve

With my best wishes
Dr. M. Abdel-Kader

Question (2): (10 Marks)

For the shown beam, using the **Moment-area method**, determine:

(a) The slope at A

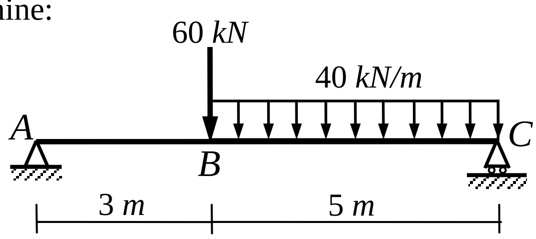
(b) The slope at B

(c) The deflection at B

and sketch the elastic curve of the beam.

$E = 200\ GPa$

$I = 290\times10^{6}\ mm^{4}$



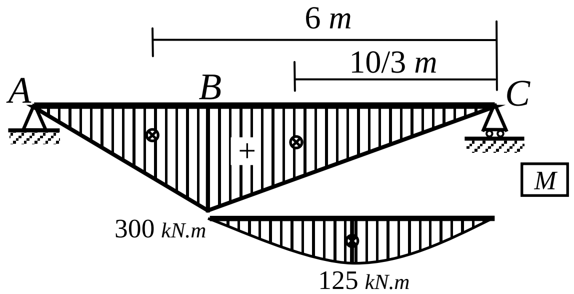
Solution:

$E = 200\ GPa = 200\times10^{6}\ kN/m^{2}$

$I = 290\times10^{6}\ mm^{4} = 290\times10^{-6}\ m^{4}$

$EI = 58000\ kN.m^{2}$

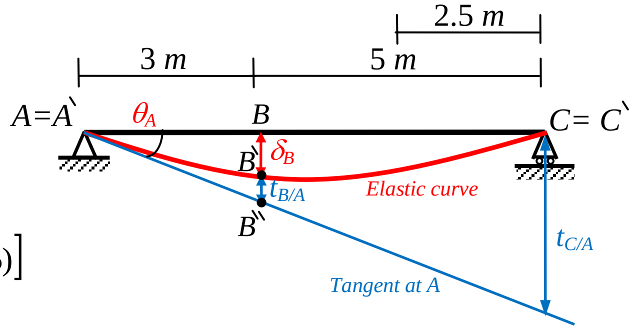
The bending moment diagram may be drawn as shown.



(a) The slope at A

$$\theta_A = \frac{t_{C/A}}{8}$$

Apply the second moment-area theorem, then



$$t_{C/A} = \frac{1}{EI}\left[Area_{AC}.\bar{X}_C\right]$$

$$= \frac{1}{EI}\left[(\tfrac{1}{2}\times3\times300)(6) + (\tfrac{1}{2}\times5\times300)(\tfrac{10}{3}) + (\tfrac{2}{3}\times5\times125)(2.5)\right]$$

$$= \frac{18725}{3EI} = \frac{18725}{3\times58000} = 749/6960 = 0.1076\ m$$

$$\therefore\ \theta_A = \frac{t_{C/A}}{8} = \frac{0.1076}{8} = 0.01345 \text{ rad} = = \ 0.77^{o}$$

$\theta_A = 0.77^{o}$ ↻

(b) The slope at B

Since the slope at A (θ_A) is known = - 0.01345, the change in slope between the tangents of the elastic curve at points B and A (θ_{BA}) is

$$\theta_{BA} = \theta_B - \theta_A = \theta_B + 0.01345$$

Apply the first moment-area theorem, then

$$\theta_{BA} = \theta_B + 0.01345 = \frac{1}{EI}\left[\text{Area of M - diagram between the points } A \text{ and } B\right]$$

$$= \frac{1}{EI}\left[Area_{AC}\right] = \frac{1}{EI}\left[(\tfrac{1}{2}\times3\times300)\right] = \frac{450}{EI} = \frac{450}{58000} = 0.00776$$

$\theta_B + 0.01345 = 0.00776$ → $\theta_B = -\ 0.00569$ rad= - 0.326^{o}

$\theta_B = 0.33^{o}$ ↻

(c) The deflection at B

The deflection at $B = \delta_B = BB'' - B'B = (3/8)\ t_{C/A} - t_{B/A}$

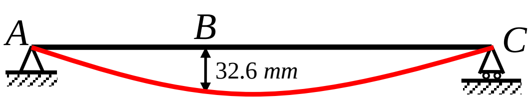
Applying the second moment-area theorem, then

$$t_{B/A} = \frac{1}{EI}\left[\text{First moment of area of M - diagram between } A \text{ and } B \text{ about } B\right]$$

$$= \frac{1}{EI}\left[Area_{AB}.\bar{X}_B\right] = \frac{1}{EI}\left[(\tfrac{1}{2}\times3\times300)(1)\right] = \frac{450}{EI} = \frac{450}{58000} = 0.00776\ m$$

$$\therefore\ \delta_B = BB'' - B'B = (3/8)\ t_{C/A} - t_{B/A} = (3/8)(0.1076) - 0.00776 = 0.03259\ m$$

$\delta_B = 32.6\ mm$ ↓



Elastic curve

With my best wishes

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