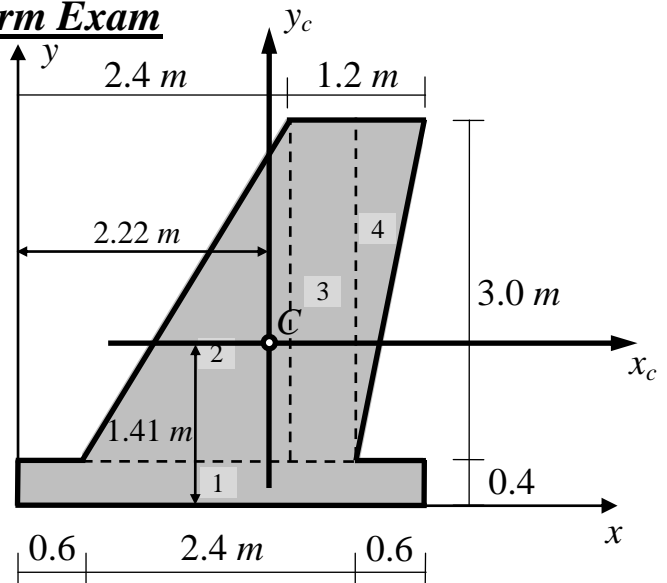


Solution of Mid-Term Exam

Question (1): (12 Marks)

For the shown cross-section, determine the following:

- The location of the centroid (\bar{x}, \bar{y}) .
- The moments of inertia about the centroidal axes (I_{x_c}, I_{y_c}) .



Solution:

$$A = 3.6 \times 0.4 + 0.5 \times 3 \times 1.8 + 3 \times 0.6 + 0.5 \times 3 \times 0.6$$

$$= \boxed{6.84 \text{ m}^2}$$

$$S_y = \sum A_i x_i = (3.6 \times 0.4)(1.8) + (0.5 \times 3 \times 1.8)(1.8) + (3 \times 0.6)(2.7) + (0.5 \times 3 \times 0.6)(3.2) = \boxed{15.192 \text{ m}^3}$$

$$S_x = \sum A_i y_i = (3.6 \times 0.4)(0.2) + (0.5 \times 3 \times 1.8)(1.4) + (3 \times 0.6)(1.9) + (0.5 \times 3 \times 0.6)(2.4) = \boxed{9.648 \text{ m}^3}$$

$$\bar{x} = \sum A_i x_i / A = 15.192 / 6.84 = \boxed{2.22 \text{ m}}$$

$$\bar{y} = \sum A_i y_i / A = 9.648 / 6.84 = \boxed{1.41 \text{ m}}$$

$$I_{x_c} = [3.6 \times 0.4^3 / 12 + (1.44)(0.2 - 1.41)^2] + [1.8 \times 3^3 / 36 + (2.7)(1.4 - 1.41)^2]$$

$$+ [0.6 \times 3^3 / 12 + (1.8)(1.9 - 1.41)^2] + [0.6 \times 3^3 / 36 + (0.9)(2.4 - 1.41)^2] = \boxed{6.592044 \text{ m}^4}$$

$$I_{y_c} = [0.4 \times 3.6^3 / 12 + (1.44)(1.8 - 2.22)^2] + [3 \times 1.8^3 / 36 + (2.7)(1.8 - 2.22)^2]$$

$$+ [3 \times 0.6^3 / 12 + (1.8)(2.7 - 2.22)^2] + [3 \times 0.6^3 / 36 + (0.9)(3.2 - 2.22)^2] = \boxed{4.122576 \text{ m}^4}$$

Question (2): (8 Marks)

A hollow steel tube with an inside diameter of 100 mm and outside diameter of 120 mm carries a tensile load P as shown in the figure.

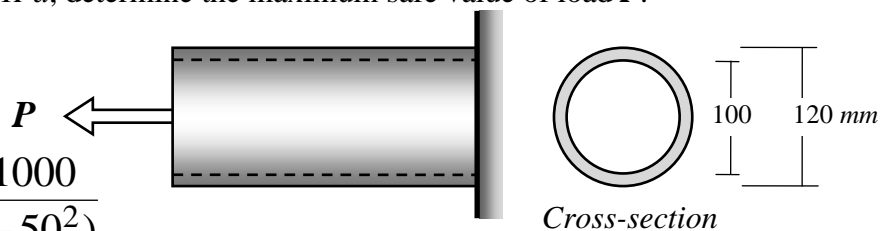
- Determine the normal stress in the tube if the load $P = 300 \text{ kN}$.
- If the allowable stress of steel is 120 MPa, determine the maximum safe value of load P .

Solution:

$$(a) \text{ Normal stress, } \sigma = \frac{P}{A} = \frac{300 \times 1000}{\pi(60^2 - 50^2)}$$

$$= 86.8 \text{ N/mm}^2 = \boxed{86.8 \text{ MPa}}$$

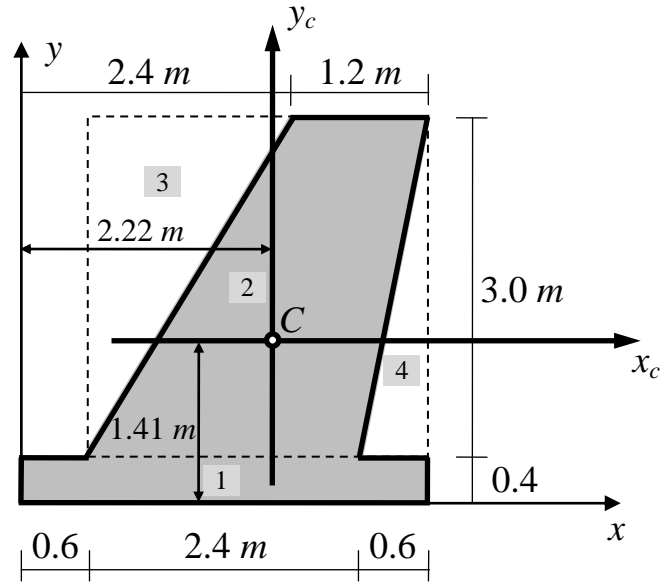
$$(b) \text{ Allowable stress, } \sigma_{all} = \frac{P_{max}}{A} \rightarrow 120 = \frac{P_{max}}{\pi(60^2 - 50^2)} \rightarrow P_{max} = 414690 \text{ N} = \boxed{414.69 \text{ kN}}$$



Question (1): (12 Marks)

For the shown cross-section, determine the following:

- (a) The location of the centroid (\bar{x}, \bar{y}) .
- (b) The moments of inertia about the centroidal axes (I_{x_c}, I_{y_c}) .



Solution:

$$A = 3.6 \times 0.4 + 3 \times 3 - 0.5 \times 3 \times 1.8 - 0.5 \times 3 \times 0.6 = \boxed{6.84 \text{ m}^2}$$

$$S_y = \sum A_i x_i = (3.6 \times 0.4)(1.8) + (3 \times 3)(2.1) - (0.5 \times 3 \times 1.8)(1.2) - (0.5 \times 3 \times 0.6)(3.4) = \boxed{15.192 \text{ m}^3}$$

$$S_x = \sum A_i y_i = (3.6 \times 0.4)(0.2) + (3 \times 3)(1.9) - (0.5 \times 3 \times 1.8)(2.4) - (0.5 \times 3 \times 0.6)(1.4) = \boxed{9.648 \text{ m}^3}$$

$$\bar{x} = \sum A_i x_i / A = 15.192 / 6.84 = \boxed{2.22 \text{ m}}$$

$$\bar{y} = \sum A_i y_i / A = 9.648 / 6.84 = \boxed{1.41 \text{ m}}$$

$$I_{x_c} = [3.6 \times 0.4^3 / 12 + (1.44)(0.2 - 1.41)^2] + [3 \times 3^3 / 12 + (9)(1.9 - 1.41)^2] - [1.8 \times 3^3 / 36 + (2.7)(2.4 - 1.41)^2] - [0.6 \times 3^3 / 36 + (0.9)(1.4 - 1.41)^2] = \boxed{6.592044 \text{ m}^4}$$

$$I_{y_c} = [0.4 \times 3.6^3 / 12 + (1.44)(1.8 - 2.22)^2] + [3 \times 3^3 / 12 + (9)(2.1 - 2.22)^2] - [3 \times 1.8^3 / 36 + (2.7)(1.2 - 2.22)^2] - [3 \times 0.6^3 / 36 + (0.9)(3.4 - 2.22)^2] = \boxed{4.122576 \text{ m}^4}$$

With my best wishes

Dr. M. Abdel-Kader