Question (1): (15 Marks)

For the shown cross-section, determine the following:

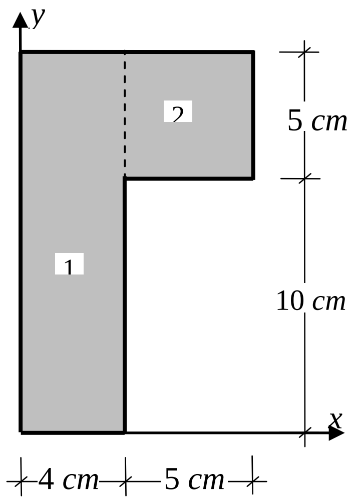
(a) The location of the centroid,

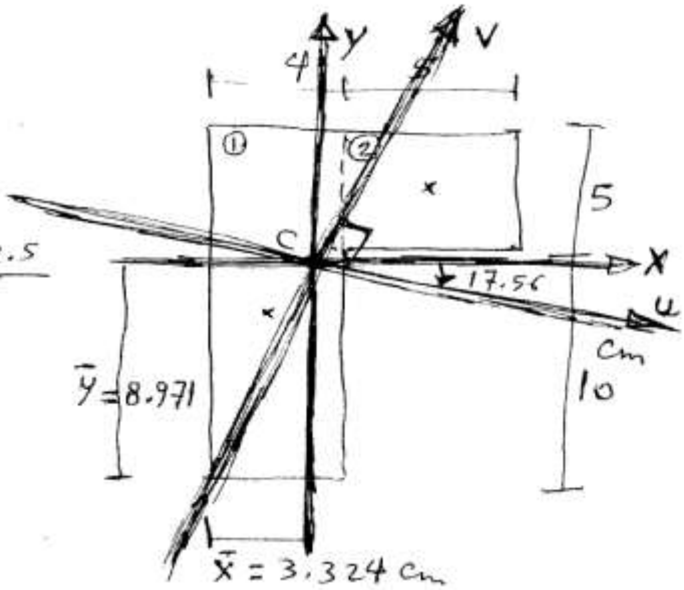
(b) The moments of inertia about the centroidal axes,

(c) The direction of the principal axes,

(d) The principal moments of inertia,

Note: *divide the cross-section to 2 elements as shown on the figure.*





(a)

$$\bar{x} = \frac{\Sigma A x}{A} = \frac{60\times 2 + 25\times 6.5}{85} = \boxed{3.324\ cm}$$

$$\bar{y} = \frac{\Sigma A y}{A} = \frac{60\times 7.5 + 25\times 12.5}{85} = \boxed{8.971\ cm}$$

(b)

$$I_x = \left[\frac{4\times 15^3}{12} + 60(-1.471)^2\right] + \left[\frac{5\times 5^3}{12} + 25(3.529)^2\right] = \boxed{1618.26\ cm^4}$$

$$I_y = \left[\frac{15\times 4^3}{12} + 60(-1.324)^2\right] + \left[\frac{5\times 5^3}{12} + 25(3.176)^2\right] = \boxed{489.44\ cm^4}$$

(c)

$$I_{xy} = \left[0 + 60(-1.471)(-1.324)\right] + \left[0 + 25(3.529)(3.176)\right] = \boxed{+397\ cm^4}$$

$$2\theta = \tan^{-1}\frac{-2(397)}{1618.26 - 489.44} = -35.12^\circ$$

$$\boxed{\theta = -17.56^\circ}$$

(d)

$$I_{u,v} = \left(\frac{1618.26 + 489.44}{2}\right) \pm \sqrt{\left(\frac{1618.26 - 489.44}{2}\right)^2 + (397)^2}$$

$$I_u = 1053.85 + 690 = \boxed{1743.85\ cm^4}$$

$$I_v = 1053.85 - 690 = \boxed{363.85\ cm^4}$$

Question (2): (15 Marks)

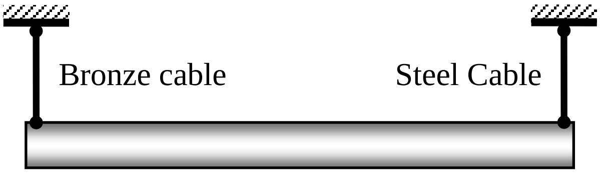
(a) Determine the smallest area of bronze and steel cables required to support the bar shown.

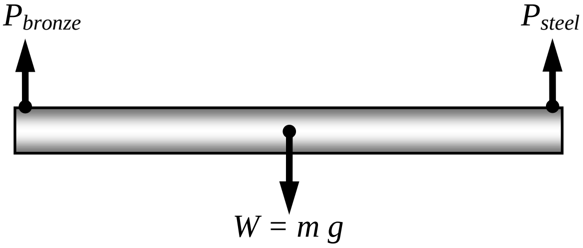
Given Data:

Mass of bar, m = 1600 kg

Maximum allowable stress for bronze, σ_{bronze} = 90 MPa

Maximum allowable stress for steel, σ_{steel} = 120 MPa





Weight of bar, $W = m\ g = (1600)(9.81) = 15696\ N$

From symmetry: $P_{bronze} = P_{steel} = \dfrac{W}{2} = \dfrac{15696}{2}$

Then $P_{bronze} = 7848\ N$ and $P_{steel} = 7848\ N$

For bronze cable:

$P_{bronze} = \sigma_{bronze}\ A_{bronze}$

$7848 = 90 \times 10^6\ A_{bronze}$ → A_{bronze}= 87.2 x$10^{-6}\ m^2$

$\boxed{A_{bronze} = 87.2\ mm^2}$

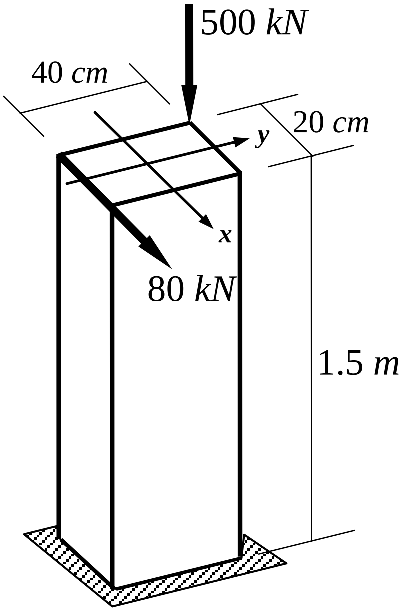
For steel cable:

$P_{steel} = \sigma_{steel}\ A_{steel}$

$7848 = 120 \times 10^6\ A_{steel}$ → A_{steel} = 65.4 x$10^{-6}\ m^2$

$\boxed{A_{steel} = 65.4\ mm^2}$

(b) A column of rectangular section carries the set of loads shown in the figure. Calculate and draw the normal stress distribution at **the base section** of the column. Neglect the column weight.



N = - 500 kN

M_x= - 500(20) = -10000 $kN.cm$

M_y=+ 500(10) – 80(150) = -7000 $kN.cm$

A= 40(20) = 800 cm^2

I_x= $20(40)^3/12$ = 106667 cm^4

I_y= $40(20)^3/12$ = 26667 cm^4

$$\sigma = \pm\frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x \quad = -\frac{500}{800} - \frac{10000}{106667} y - \frac{7000}{26667} x$$

N.A.

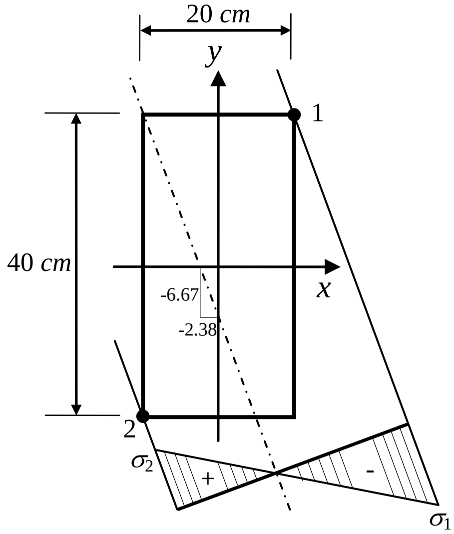
$$-\frac{500}{800} - \frac{10000}{106667} y - \frac{7000}{26667} x = \mathbf{0}$$

At x = 0; $y = -\dfrac{500 \times 106667}{800 \times 10000}$ = 6.67 cm → (0, 6.67)

At y = 0; $x = -\dfrac{500 \times 26667}{800 \times 7000}$ = 2.38 cm → (2.38, 0)

$$\sigma_1 = -\frac{500}{800} - \frac{10000}{106667}(20) - \frac{7000}{26667}(10) = \boxed{-5.12\ kN/cm^2}$$

$$\sigma_2 = -\frac{500}{800} - \frac{10000}{106667}(-20) - \frac{7000}{26667}(-10) = \boxed{+3.87\ kN/cm^2}$$



Normal stress distribution

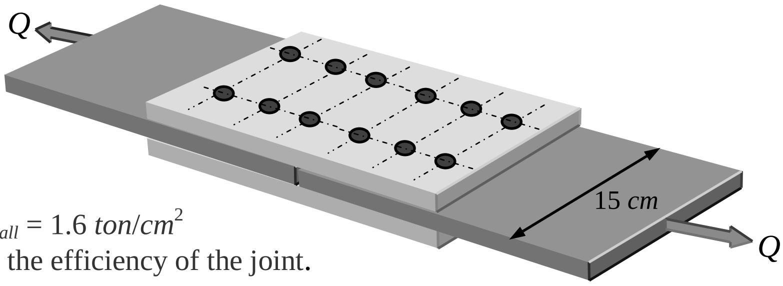
Question (3): (15 Marks)

(a) A bolted butt joint is shown. The diameter of the bolts is 1.8 *cm*. The width of the plates is 15 *cm*, and the thickness of the plates is 1.2 *cm*. The allowable stresses are as follows:

Bolts: $\tau_{all} = 1.1\ ton/cm^2$,

Plates: $\sigma_{t\ all} = 1.4\ ton/cm^2$ and $\sigma_{bearing\ all} = 1.6\ ton/cm^2$

Determine the strength of the joint, and the efficiency of the joint.



1. Failure due to bolt shear:

$$Q_{bolt\ shear} = n\,\tau_{all}\left(\frac{\pi\, d^2}{4}\right) = 12(1.1)\left(\frac{\pi\, 1.8^2}{4}\right) = 33.59\ ton \qquad \text{------- (1)}$$

2. Bearing failure of bolt or plate (compression failure):

$$Q_{bearing} = n\,(d\,t)\,\sigma_{bearing\ \ all} = 6\ (1.8 \times 1.2)(1.6) = 20.74\ ton \qquad \text{------- (2)}$$

3. Plate tearing:

$$Q_{tearing} = (w - n\,d)\,t\,\sigma_{t\ all} = (15 - 2\times1.8)(1.2)(1.4) = 19.15\ ton \qquad \text{------- (3)}$$

From 1-3, the joint fails first in plate tearing at **the lowest load of 19.15 *ton***.

Plate Strength = (the complete area of plate cross section) times (the allowable tensile stress for plate material),

$$Q_{plate} = w\,t\,\sigma_{t\ all} = (15)(1.2)(1.4) = 25.2\ ton$$

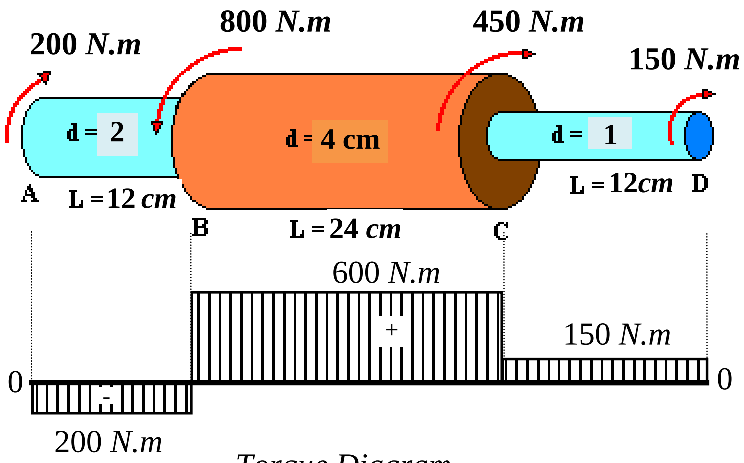
Efficiency = Joint Strength / Plate Strength = 19.15 / 25.2 = 0.76 = **76 %**

(b) A solid compound shaft is subjected to torques as shown. The shaft is in rotational equilibrium.

i) Determine the maximum transverse shear stress in each section of the shaft due to the applied torque.

ii) Determine the angle of twist of end **D** with respect to end **A**.

$G = 8000\ kN/cm^2$.



Torque Diagram

Part AB

T = 200 N.m = 20000 N.cm; r = 1 cm

$J = I_P = \pi\,(r)^4/2 = \pi\,(1)^4/2 = 1.57\ cm^4$

So, $\tau = \dfrac{Tr}{J} = \dfrac{(20000)(1)}{1.57}$

$= 12739\ N/cm^2 =$ **$12.74\ kN/cm^2$**

Part BC

T = 600 N.m = 60000 N.cm; r = 2 cm

$J = I_P = \pi\,(r)^4/2 = \pi\,(2)^4/2 = 25.133\ cm^4$

So, $\tau = \dfrac{Tr}{J} = \dfrac{(60000)(2)}{25.133} = 4775\ N/cm^2 =$ **$4.78\ kN/cm^2$**

Part CD

T = 150 N.m = 15000 N.cm; r = 0.5 cm

$J = I_p = \pi\,(r)^4/2 = \pi\,(0.5)^4/2 = 0.098\ cm^4$

So, $\tau = \dfrac{Tr}{J} = \dfrac{(15000)(0.5)}{0.098} = 76530\ N/cm^2 =$ **$76.5\ kN/cm^2$**

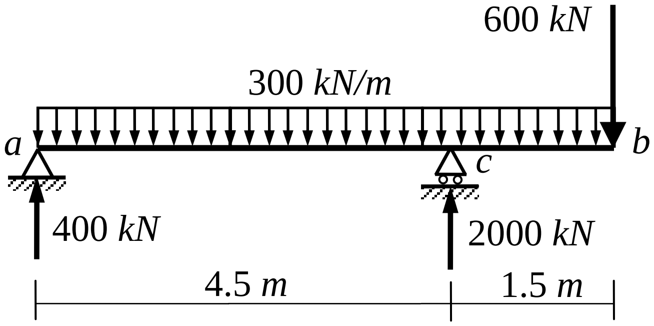
$$\phi_{D/A} = \frac{15000 \times 12}{0.098 \times 8.0 \times 10^6} + \frac{60000 \times 24}{25.133 \times 8.0 \times 10^6} + \frac{-20000 \times 12}{1.57 \times 8.0 \times 10^6} = 0.23 + 0.007 - 0.019 = 0.218\ \text{rad} = 12.5^o$$

Question (4): (15 Marks)

For the shown beam, calculate and draw:

(a) The **normal** stress distribution over the cross-section at ***c***.

(b) The **shear** stress distribution over the cross-section at ***a***.



(a) M=1237.5 kN.m=123750 kN.cm

$$\sigma_t = \sigma_c = \frac{M}{I}\,y = \frac{123750}{115000}(20) = 21.52 \text{ kN/cm}^2$$

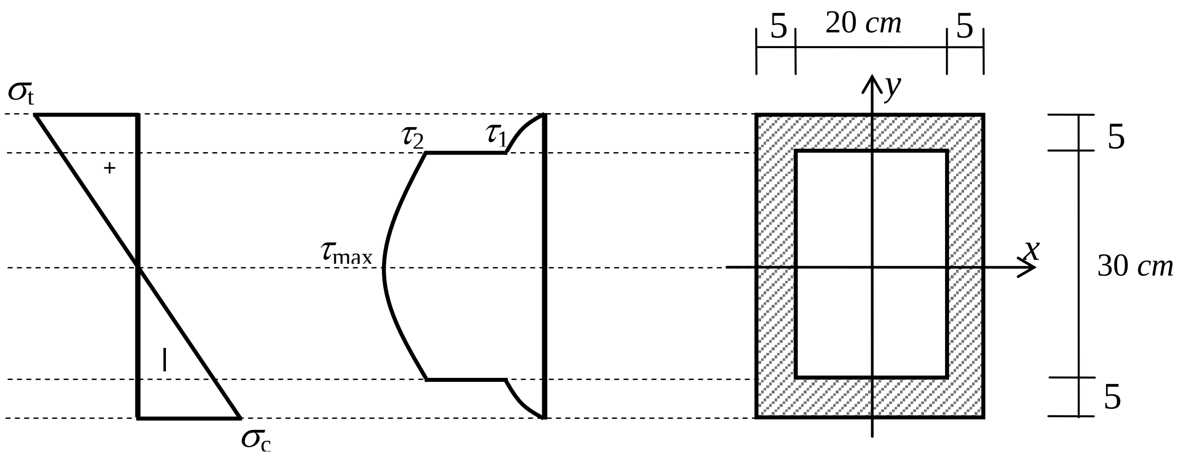
$$I = \frac{30\times 40^3}{12} - \frac{20\times 30^3}{12} = 115000 \text{ cm}^4$$

(b) Q = 400 kN

$$\tau_1 = \frac{QS}{Ib} = \frac{(400)(30\times 5)(17.5)}{115000\times 30} = 0.304 \text{ kN/cm}^2$$

$$\tau_2 = \frac{QS}{Ib} = \frac{(400)(30\times 5)(17.5)}{115000\times 10} = 0.913 \text{ kN/cm}^2$$

$$\tau_{max} = \frac{QS}{Ib} = \frac{(400)(30\times 5\times 17.5 + 2\times 15\times 5\times 7.5)}{115000\times 10} = 1.304 \text{ kN/cm}^2$$



Normal Stress Distribution

Shear Stress Distribution

Cross-section of the beam

With my best wishes

Dr. M. Abdel-Kader