Ministry of Higher Education
Giza Higher Institute of Engineering & Technology
Civil Engineering Department
Course Name: **Theory of Structures (2)**
Course Code : **CIV 202** Date : **9 / 5 / 2018**

Academic Year : **2017/2018**
Semester : **Second**
Level : **2rd**
Time : **3 Hours**
Examiners: ***Dr. M. Abdel-Kader***

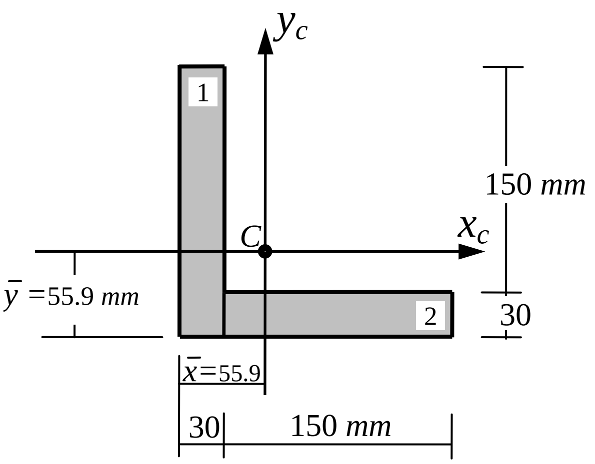
Answer of Final Exam

Total Marks: **60** No. of Questions:**6** (Attempt all questions)

Question (1): (10 Marks)

For the shown cross-section, determine the following:

(a) The location of the centroid.
(b) The moments of inertia about the centroidal axes.
(c) The direction of the principal axes.
(d) The principal moments of inertia.



Solution:

(a) Location of the centroid:

$$\bar{x} = \frac{\Sigma A_i\, x_i}{A} = \frac{(180\times30)(15)+(150\times30)(105)}{180\times30+150\times30} = \boxed{55.9\ mm}$$

$$\bar{y} = \frac{\Sigma A_i\, y_i}{A} = \frac{(180\times30)(90)+(150\times30)(15)}{180\times30+150\times30} = \boxed{55.9\ mm}$$

(b) Moments of inertia about the centroidal axes:

$$I_{x_c} = \left[\frac{30\times180^3}{12} + (180\times30)(34.1)^2\right] + \left[\frac{150\times30^3}{12} + (150\times30)(\text{-}40.9)^2\right]$$

$$= \boxed{28724319\ mm^4}$$

$$I_{y_c} = \left[\frac{180\times30^3}{12} + (180\times30)(\text{-}40.9)^2\right] + \left[\frac{30\times150^3}{12} + (150\times30)(49.1)^2\right] = \boxed{28724319\ mm^4}$$

Note that the section is symmetrical about an axis makes an angle =45° with the horizontal, therefore, you can calculate only $\bar{x}$ and I_{x_c}, and then $\bar{y} = \bar{x}$ & $I_{y_c} = I_{x_c}$.

(c) Direction of the principal axes:

$$I_{x_c y_c} = [0 + (180\times30)(\text{-}40.9)(34.1)] + [0 + (150\times30)(49.1)(\text{-}40.9)] = \boxed{-16568181\ mm^4}$$

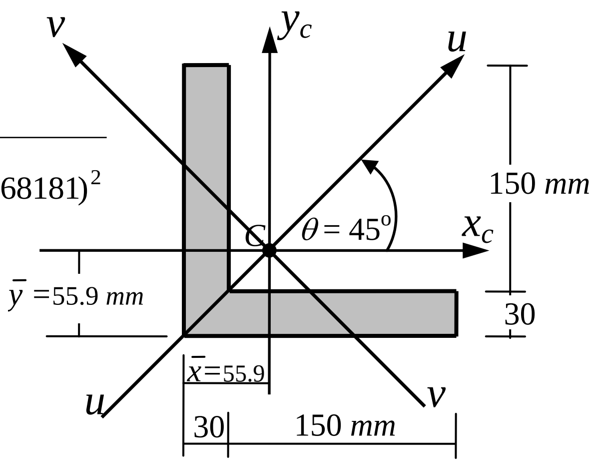
$$\tan 2\theta = \frac{-2I_{xy}}{(I_x - I_y)} = \frac{-2(\text{-}16568181)}{(28724319 - 28724319)} = \infty \quad \text{so,}\quad 2\theta = 90^o \rightarrow \boxed{\theta = 45^o}$$

(d) Principal moments of inertia:

$$I_{\substack{u\\v}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$I_{\substack{u\\v}} = \frac{28724319+28724319}{2} \pm \sqrt{\left(\frac{28724319-28724319}{2}\right)^2 + (-16568181)^2}$$

$$\boxed{\begin{aligned} I_u &= 45292500\ mm^4 = 45.29\times10^6\ mm^4 \\ I_v &= 12156138\ mm^4 = 12.16\times10^6\ mm^4 \end{aligned}}$$



With best wishes
Dr. M. Abdel-Kader

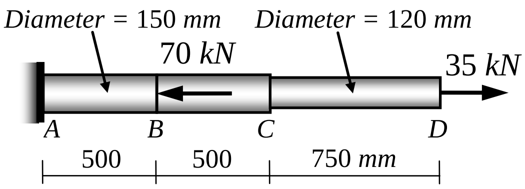
Question (2): (10 Marks)

A stepped bar is subjected to axial loads as shown. Calculate the following:

(a) The normal stress in each part.

(b) The total elongation.

Where $E = 2.32\ GPa$



Solution:

(a) σ in each part

For part AB:

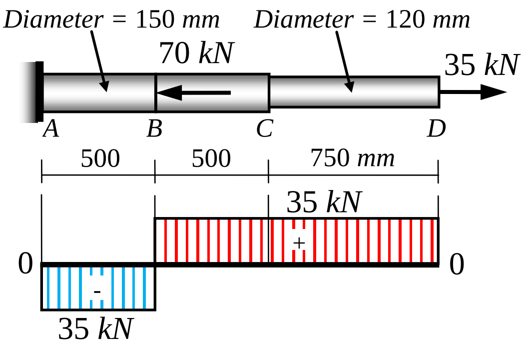
$$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{-35000}{\pi(75)^2} = -1.981\,MPa$$

For part BC:

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{35000}{\pi(75)^2} = 1.981\,MPa$$

For part CD:

$$\sigma_{CD} = \frac{P_{CD}}{A_{CD}} = \frac{35000}{\pi(60)^2} = 3.095\,MPa$$



Normal Force Diagram

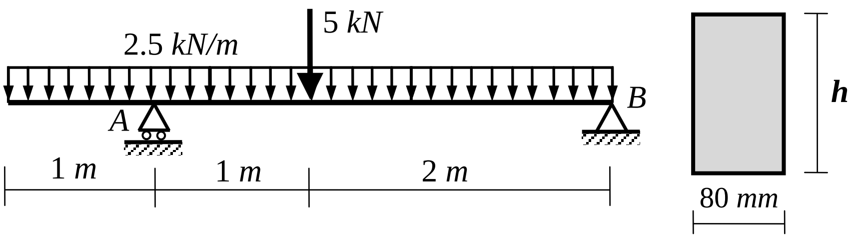
(b) Total elongation δ

$$\delta = \sum\frac{PL}{EA} = \sum\frac{P}{A}\frac{L}{E} = \sum\frac{\sigma L}{E} = \frac{1}{E}\sum\sigma L = \frac{1}{2320}[-1.981\times500+1.981\times500+3.095\times750] = 1.00\,mm$$

Question (3): (10 Marks)

Determine the minimum height h of the cross section of the beam loaded as shown. The maximum flexural stress, $f_{b\,max} = 20\ MPa$.

Note: *S.F.D* and *B.M.D* are required.



Solution:

$$+\circlearrowleft\sum M_B = 0:$$

$$A_y(3) - (2.5\times4)(2) - 5(2) = 0 \rightarrow A_y = 10\ kN \uparrow$$

$$+\uparrow\sum F_y = 0:$$

$$A_y + B_y - (2.5\times4) - 5 = 0 \rightarrow B_y = 5\ kN \uparrow$$

$$I_x = \frac{80h^3}{12} = \tfrac{20}{3}h^3\ mm$$

$$M_{max} = 5\,kN.m = 5(1000)(1000)\ N.mm$$

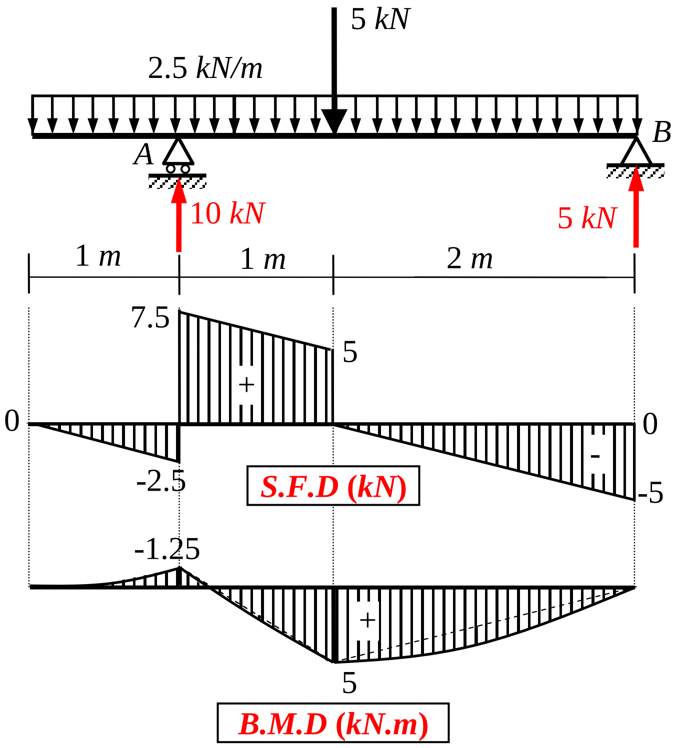
$$= 5\times10^6\ N.mm$$

$$y_{max} = \tfrac{1}{2}h\ \ mm$$

$$f_{b\,max} = \frac{M_x}{I_x}y_{max} \rightarrow 20 = \frac{5\times10^6}{\frac{20}{3}h^3}(\tfrac{1}{2}h)$$

$$\rightarrow h^2 = 18750\ mm^2 \rightarrow h = 136.93\ mm$$

$$\therefore\ \boxed{h = 137\ mm}$$

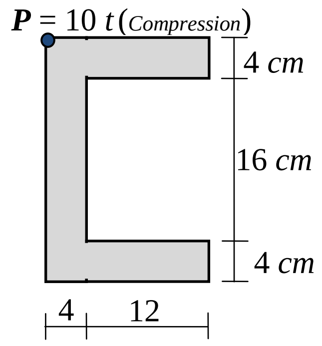


With best wishes

Dr. M. Abdel-Kader

Question (4): (10 Marks)

A cross-section is subjected to axial compressive load ***P*** as shown, calculate and draw the normal stress distribution over the cross-section.



Solution:

$A = (4\times12)\times2+(4\times24) = 192\ cm^2$

$$\bar{x} = \frac{\sum A_i x_i}{A} = \frac{2(4\times12)(8))}{192} = 4\,cm$$

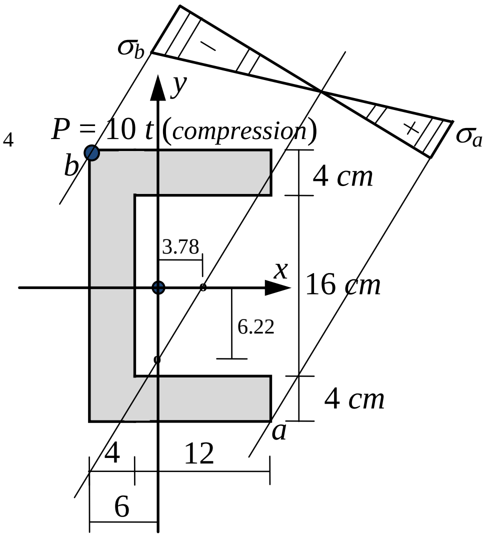
$$I_{x_c} = 2\left[\frac{12\times4^3}{12} + (12\times4)(10)^2\right] + \left[\frac{4\times24^3}{12}\right] = 14336\ cm^4$$

$$I_{y_c} = 2\left[\frac{4\times12^3}{12} + (4\times12)(4)^2\right] + \left[\frac{24\times4^3}{12} + (24\times4)(4)^2\right] = 4352\ cm^4$$

$N = -10\ t$

$M_x = -10\times12 = -120\ t.cm$

$M_y = 10\times6 = 60\ t.cm$



$$\sigma = \pm\frac{N}{A} \pm \frac{M_x}{I_x}y \pm \frac{M_y}{I_y}x = -\frac{10}{192} - \frac{120}{14336}y + \frac{60}{4352}x$$

N.A.

$$-\frac{10}{192} - \frac{120}{14336}y + \frac{60}{4352}x = 0$$

At $x = 0$; $y = -\frac{10\times14336}{192\times120} = -6.22\ cm$ → (0, -6.22)

At $y = 0$; $x = +\frac{10\times4352}{192\times60} = +3.78\ cm$ → (-3.78, 0)

$$\sigma_a = -\frac{10}{192} - \frac{120}{14336}(-12) + \frac{60}{4352}(10) = 0.186 = \boxed{+0.19\ t/cm^2}$$

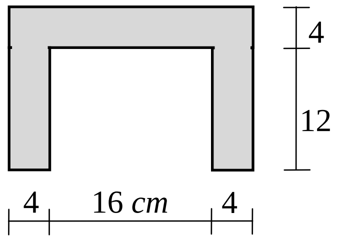
$$\sigma_b = -\frac{10}{192} - \frac{120}{14336}(12) + \frac{60}{4352}(-6) = -0.235 = \boxed{-0.24\ t/cm^2}$$

With best wishes

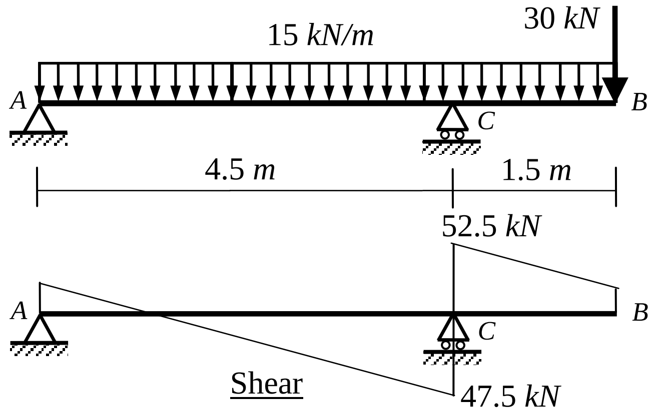
Dr. M. Abdel-Kader

Question (5): (10 Marks)

For the shown beam, calculate and draw the shear stress distribution over the cross-section at ***C***.



Cross-section of the beam



Solution:

From the solution of Question (4), the location of the centroid of the section is known and

$I_x \ here = I_{y_c} = 4352 \ cm^4$

The reaction at the support A =20 kN and at the support C = 100 kN

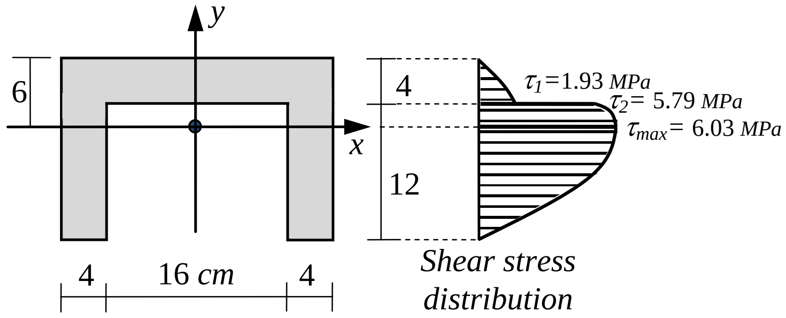
The shear force at section just right to C, Q = 30 + 15×1.5 = 52.5 kN

The shear force at section just left to C, Q = 100-52.5 = 47.5 kN

$$\tau_1 = \frac{QS}{Ib} = \frac{(52.5)(24\times4)(4)}{4352\times(24)} = 0.193 \ kN/cm^2$$

$$\tau_2 = \frac{QS}{Ib} = \frac{(52.5)(24\times4)(4)}{4352\times(4+4)} = 0.579 \ kN/cm^2$$

$$\tau_{\max} = \frac{QS}{Ib} = \frac{(52.5)[(24\times4)(4)+2(2\times4)\times1]}{4352\times(4+4)} = 0.603 \ kN/cm^2$$

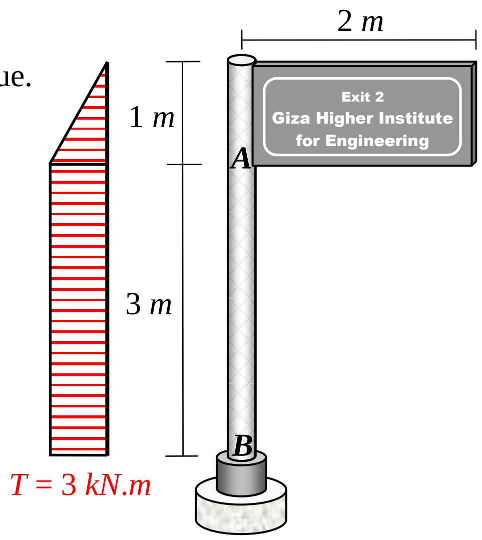


Question (6): (10 Marks)

A tube must resist a torque due to the wind load on the sign board. The wind pressure is 1.5 kN/m^2. The tube has an outside diameter of 200 mm and a thickness of 5 mm. Determine:

(a) The maximum shear stress τ_{max} in the tube at section ***B*** due to the torque.

(b) The relative angle of twist between ***A*** and ***B***, where G = 30 GPa



Solution:

(a) The maximum shear stress τ_{max}

The torque due wind load, T = wind load × arm

= [1.5(2×1)] × 1

= 3 $kN.m$ = 3×10^6 $N.mm$

The polar moment of inertia, $J = I_P$ for a hollow tube is:

$$J = \frac{\pi\left(r_o^4 - r_i^4\right)}{2} = \frac{\pi\left(100^4 - 95^4\right)}{2} = 29.14\times10^6 \ mm^4$$

Therefore, the maximum shear stress

$$\tau_{\max} = \frac{Tr}{J} = \frac{3\times10^6(100)}{29.14\times10^6} = 10.3 \ MPa$$

The maximum shear stress is at the outside surface of the tube and has a magnitude of 10.3 MPa.

(b) The relative angle of twist between ***A*** and ***B***:

$$\phi_{A/B} = \frac{TL}{JG} = \frac{3\times10^6(3000)}{29.14\times10^6\left(30\times10^3\right)} = 0.0103\text{rad} = 0.59^\circ$$

With best wishes

Dr. M. Abdel-Kader