Ministry of Higher Education **Giza Higher Institute for Eng. & Tech.** **Civil Engineering Department**	Academic Year : **2016/2017** Semester : **Second** Level : **2nd**	
Course Name: **Theory of Structures (2)**	Time : **3 Hours**	
Course Code : **CIV 202** — Date : **28 / 5 / 2017**	Examiner: ***Dr. M. Abdel-Kader***	
Answer of Final Term Exam		
Total Marks: **60**	No. of Questions: **5**	

Question (1): (12 Marks)

For the shown cross-section, determine the following:

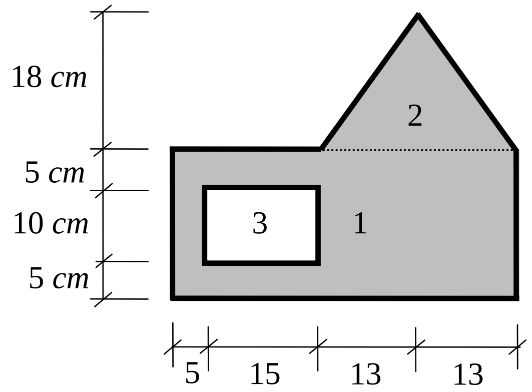
(a) The location of the centroid.

(b) The moments of inertia about the centroidal axes (I_{x_c} & I_{y_c}).

(c) The direction of the principal axes.

(d) The principal moments of inertia.

Note: *Divide the cross-section to 3 elements as shown on the figure.*



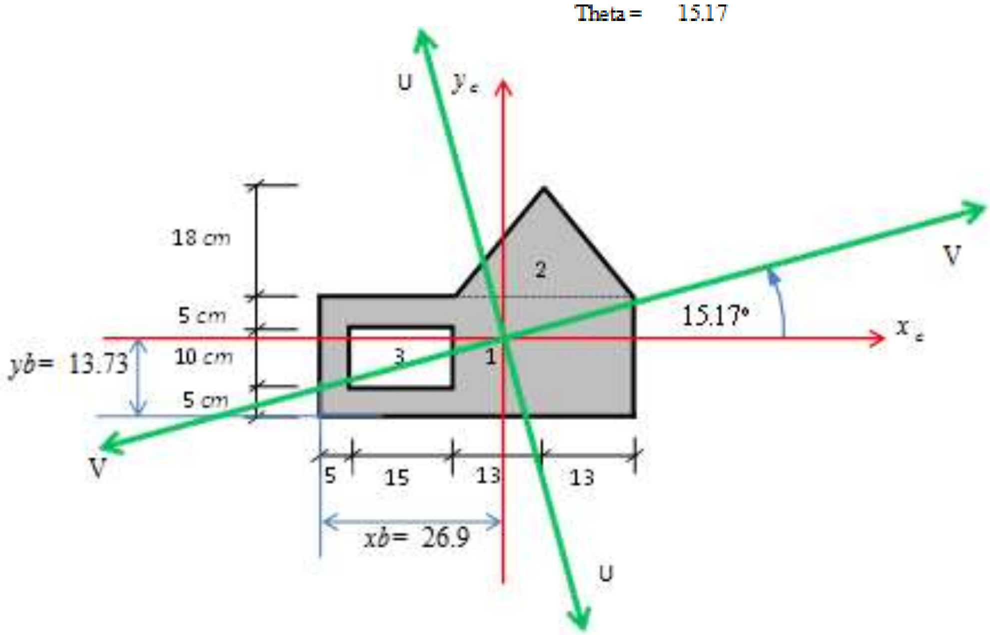
No.	b	h	A	x	y	Ax	Ay	x-xb	y-yb	I_x	$A(y-yb)^2$	I_y	$A(x-xb)^2$	Ixcyc	I_{xy}
1	46	20	920.00	23.00	10.00	21160.00	9200.00	-3.90	-3.73	30666.67	12793.58	162226.67	13988.91	0.00	13377.90
2	26	18	234.00	33.00	26.00	7722.00	6084.00	6.10	12.27	4212.00	35234.64	6591.00	8708.85	0.00	17517.22
3	15	10	-150.00	12.50	10.00	-1875.00	-1500.00	-14.40	-3.73	-1250.00	-2085.91	-2812.50	-31101.42	0.00	-8054.49
			1004.00			27007.00	13784.00			33628.67	45942.31	166005.17	-8403.66		22840.64

a) $xb=$ 26.90 cm; $yb=$ 13.73 cm

b) $I_x=$ 79570.98 cm^4; $I_y=$ 157601.51 cm^4

c) tan (2 Theta) 0.585428; 2 Theta = 30.35; Theta = 15.17

d) $I_u=$ 163795.59 cm^4; $I_v=$ 73376.89 cm^4



$A_1 = 20\times46 = 920\ cm^2$
$A_2 = \frac{1}{2}\times26\times18 = 234\ cm^2$
$A_3 = 10\times15 = 150\ cm^2$
$\rightarrow A = 920+234-150 = 1004\ cm^2$

$\bar{X} = \frac{920(23)+234(33)-150(12.5)}{1004} = \frac{27007}{1004} = \boxed{26.9\ cm}$

$\bar{Y} = \frac{920(10)+234(26)-150(10)}{1004} = \frac{13784}{1004} = \boxed{13.73\ cm}$

$I_{xy} = [0+920(23-26.9)(10-13.73)]+[0+234(33-26.9)(26-13.73)] - [0+150(12.5-26.9)(10-13.73)] = \boxed{22840.6\ cm^4}$

$\tan 2\theta = \frac{-2I_{xy}}{I_x-I_y} = \frac{-2(22840.6)}{79571-157601.5} = 0.5854$

$2\theta = 30.346$ $\quad \boxed{\theta = 15.17^\circ}$

$I_x = \left[\frac{46\times20^3}{12}+920(10-13.73)^2\right]+\left[\frac{26\times18^3}{36}+234(26-13.73)^2\right] - \left[\frac{15\times10^3}{12}+150(10-13.73)^2\right]$ $43466.5+39447.4-3336.9$ $= \boxed{79571\ cm^4}$

$I_y = \left[\frac{20\times46^3}{12}+920(23-26.9)^2\right]+\left[\frac{18\times26^3}{48}+234(33-26.9)^2\right] - \left[\frac{10\times15^3}{12}+150(12.5-26.9)^2\right]$ $176215.9+15299.8-33914.5$ $= \boxed{157601.5\ cm^4}$

$I_{u,v} = \left(\frac{79571+157601.5}{2}\right) \pm \sqrt{\left(\frac{79571-157601.5}{2}\right)^2+(22840.6)^2}$
$= 118586.25 \pm 45209.3$

$\boxed{I_u = 163796\ cm^4}$
$\boxed{I_v = 73377\ cm^4}$

With my best wishes
Dr. M. Abdel-Kader

Question (2): (12 Marks)

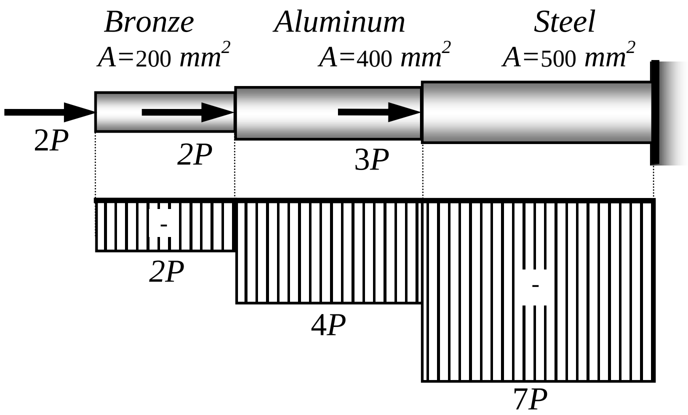
A rod of variable cross-section is subjected to axial loads as shown. Determine the maximum safe value of axial load P.

Given Data:

Allowable stress for bronze = 100 *MPa*

Allowable stress for aluminum = 90 *MPa*

Allowable stress for steel = 140 *MPa*



Normal Force Diagram

For bronze:

$$\sigma_{bronze} = \frac{P_{bronze}}{A_{bronze}} \le 100 \times 10^6 \quad N/m^2$$

$$\frac{2P}{200 \times 10^{-6}} \le 100 \times 10^6 \qquad \therefore \; P \le 10000\, N \ldots(1)$$

For aluminum:

$$\sigma_{alum} = \frac{P_{alum}}{A_{alum}} \le 90 \times 10^6\, N/m^2 \rightarrow \frac{4P}{400 \times 10^{-6}} \le 90 \times 10^6 \qquad \therefore \; P \le 9000\, N \ldots(2)$$

For steel:

$$\sigma_{steel} = \frac{P_{steel}}{A_{steel}} \le 140 \times 10^6\, N/m^2 \rightarrow \frac{7P}{500 \times 10^{-6}} \le 140 \times 10^6 \qquad \therefore \; P \le 10000\, N \ldots(3)$$

Form (1), (2) and (3), the maximum safe value of axial load P = 9000 N = 9 kN

$$\boxed{P_{Safe} = 9\ kN}$$

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Question (3): (12 Marks)

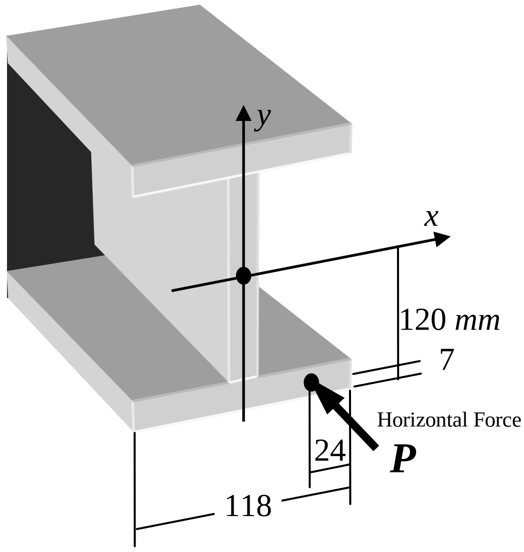
A horizontal load ***P*** is applied as shown to a I-section. The properties of the I-section are:

Area, $A = 4806\ mm^2$

Section Modulus, $Z_x = 406\times10^3\ mm^3$

Section Modulus, $Z_y = 48\times10^3\ mm^3$

Determine the largest permissible load ***P*** if the compressive stress in the member is not to exceed 80 *MPa*. Neglect the member weight.



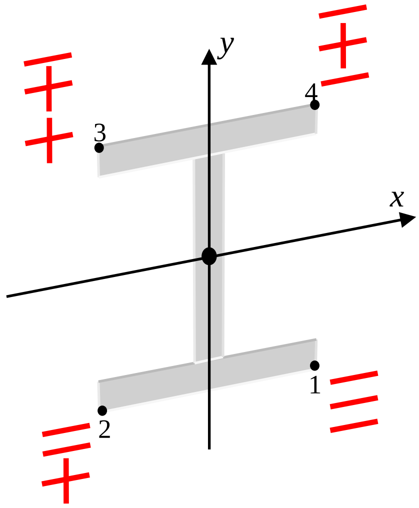
Solution:

$N = -\boldsymbol{P}$

$M_x = +120\ \boldsymbol{P}$

$M_y = -35\ \boldsymbol{P}$

$$\sigma = \pm\frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x \quad \textbf{or} \quad \sigma = \pm\frac{N}{A} \pm \frac{M_x}{z_x} \pm \frac{M_y}{z_y}$$



The sign of each component can be determine by carefully examining the sketch of the force-moment system as shown. It can be seen that the maximum compressive stress occurs at point 1; the normal force, the bending moment about x-axis and the bending moment about y-axis, all cause compression at this point. So,

$$\text{Normal stress at point } 1 = \sigma_1 = -\frac{N}{A} - \frac{M_x}{z_x} - \frac{M_y}{z_y}$$

$$\sigma_1 = -\frac{P}{4806} - \frac{120P}{406000} - \frac{35P}{48000} = -1.2328\times10^{-3} P$$

And this maximum value of compressive stress at point 1 (σ_1) should be not to exceed 80 *MPa*. So,

$$1.2328\times10^{-3} P \leq 80\ N/mm^2 \quad \rightarrow \quad P \leq 64893N$$

$\therefore$ The largest permissible load ***P*** = 64893 *N* = 64.9 *kN*

$\boldsymbol{P_{max}} = 64.9\ kN$

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Question (4): (12 Marks)

For the shown column of variable circular cross-section,

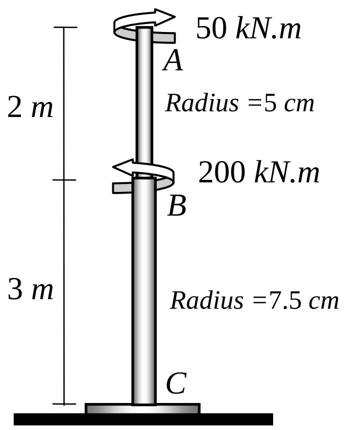
(a) Draw the twisting moment diagram.

(b) Determine the maximum shear stress in each part (***AB*** and ***BC***).

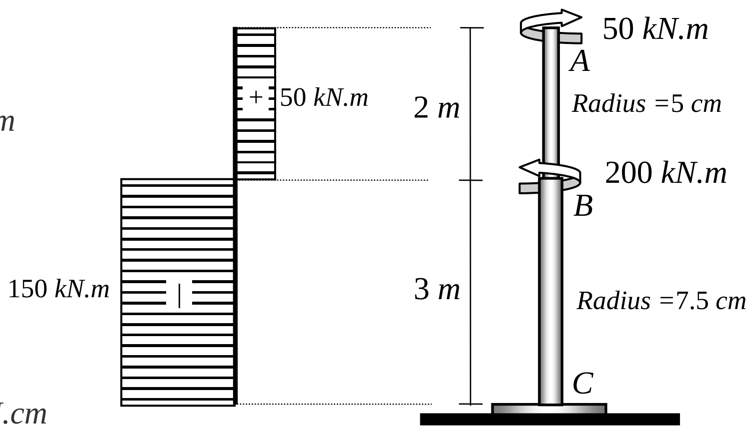
(c) Determine the angle of twist ϕ of section *A* with respect to the fixed support at *C*.

G = 8000 kN/cm^2

$$\tau = \frac{Tr}{J} \quad \text{and} \quad \phi = \frac{TL}{JG}$$



(a) The twisting moment diagram is as shown



Twisting Moment Diagram

(b)

Part *AB*

Twisting moment = T = 50 $kN.m$ = 5000 $kN.cm$

$J = \pi r^4/2 = \pi (5)^4/2 = 981.75\ cm^4$

So, $\tau = \dfrac{Tr}{J} = \dfrac{(5000)(5)}{981.75} = 25.46\ kN/cm^2$

= **25.46 kN/cm^2** = **254.6 MPa**

Part *BC*

Twisting moment = T = 150 $kN.m$ = 15000 $kN.cm$

$J = \pi r^4/2 = \pi (7.5)^4/2 = 4970\ cm^4$

So, $\tau = \dfrac{Tr}{J} = \dfrac{(15000)(7.5)}{4970}$

= 22.64 kN/cm^2 = **22.64 kN/cm^2** = **226.4 MPa**

(c)

$$\phi_{A/C} = \phi_{A/B} + \phi_{B/C} = \frac{5000 \times 200}{981.75 \times 8000} + \frac{-15000 \times 300}{4970 \times 8000} = 0.1273 - 0.1132 = 0.0141 \text{ rad} = \boxed{0.82^{o}}$$

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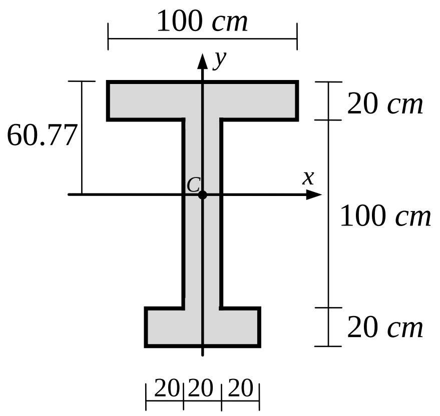
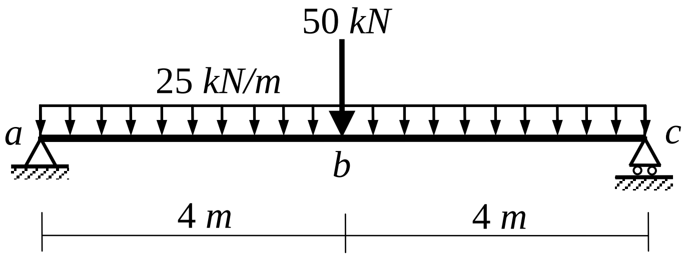
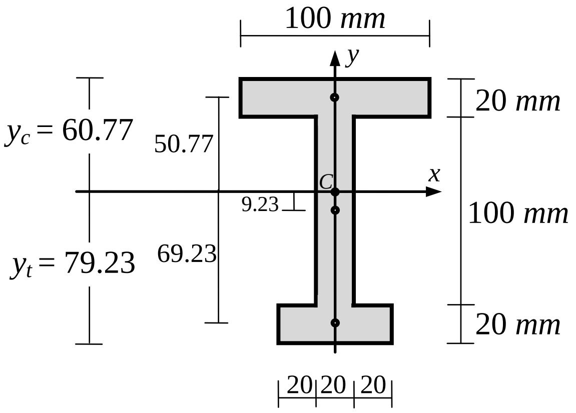
Question (5): (12 Marks)

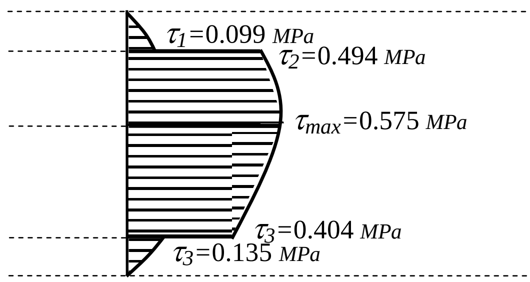
For the shown beam, calculate and draw the shear stress distribution over the cross-section at ***a***.



Cross-section of the beam







Shear stress distribution

$I_x = [60\times20^3/12 + (1200)(-69.23)^2] + [20\times100^3/12 + (2000)(-9.23)^2] + [100\times20^3/12 + (2000)(50.77)^2]$
$= 12850256\ cm^4 = 12.85\times10^6\ cm^4$

Shear force at a, $Q_a = wL/2 + P/2 = 25\times8/2 + 50/2 = 100+25 = 125\ kN = 125\times10^3\ N$

Shear stress, $\tau_1 = (Q_a S_x) / (I_x b) = (125\times10^3 \times 2000\times50.77) / (12.85\times10^6\times100)$
$= 9.877\ N/cm^2 = 0.099\ MPa$ — $\boxed{\tau_1 = 0.099\ MPa}$

Shear stress, $\tau_2 = (125\times10^3 \times 2000\times50.77) / (12.85\times10^6\times20)$
$= 49.387\ N/cm^2 = 0.494\ MPa$ — $\boxed{\tau_2 = 0.494\ MPa}$

Shear stress, $\tau_{max} = (125\times10^3)(2000\times50.77+20\times40.77^2/2) / (12.85\times10^6\times20)$
$= 57.472\ N/cm^2 = 0.575\ MPa$ — $\boxed{\tau_{max} = 0.575\ MPa}$

Shear stress, $\tau_3 = (125\times10^3)(1200\times69.23) / (12.85\times10^6\times20)$
$= 40.41\ N/cm^2 = 0.404\ MPa$ — $\boxed{\tau_3 = 0.404\ MPa}$

Shear stress, $\tau_4 = (125\times10^3)(1200\times69.23) / (12.85\times10^6\times60)$
$= 13.4689\ N/cm^2 = 0.135\ MPa$ — $\boxed{\tau_4 = 0.135\ MPa}$

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