Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (2)**
Course Code : **CIV 202**

Academic Year : **2015–2016**
Semester : **Second**
Level : **2nd**
Time : **3 Hours**
Date : **28 / 5 / 2016**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Second Semester Final Exam

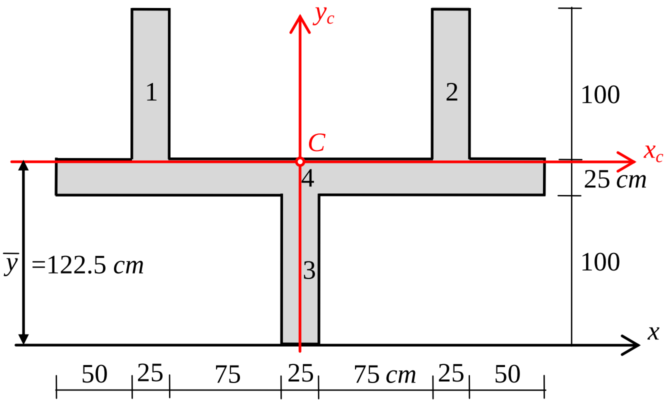
Question (1): (12 Marks)

For the shown cross-section, determine the following:

(a) The location of the centroid.

(b) The moments of inertia about the centroidal axes (I_{x_c} & I_{y_c}).

Note: *Divide the cross-section to 4 elements as shown on the figure.*



Element	A_i	y_i	$A_i\,y_i$	I_{xi}	$dy_i=y_i-\bar{y}$	$A_i\,(dy_i)^2$	I_{yi}	dx_i	$A_i\,(dx_i)^2$
1	25×100	175	437500	$25\times100^3/12$	52.5	6890625	$100\times25^3/12$	-100	25000000
2	25×100	175	437500	$25\times100^3/12$	52.5	6890625	$100\times25^3/12$	100	25000000
3	25×100	50	125000	$25\times100^3/12$	-72.5	13140625	$100\times25^3/12$	0	0
4	25×325	112.5	914062.5	$325\times25^3/12$	-10	812500	$25\times325^3/12$	0	0
Σ	15625	--	1914062.5	6673177	--	27734375	71907552		50000000

(a) Location of the centroid:

- From symmetry y_c-axis is as shown

- $$\bar{y} = \frac{\sum A_i\, y_i}{\sum A_i} = \frac{1914062.5}{15625} = \boxed{122.5\ cm}$$

(b) The moments of inertia about the centroidal axes

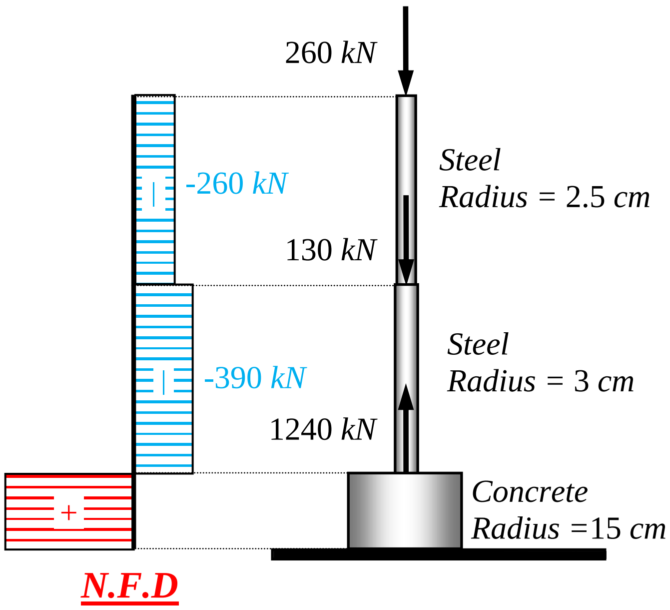
$$I_{x_c} = \sum\left(I_{x_i} + A_i(dy_i)^2\right) = \sum I_{x_i} + \sum A_i(dy_i)^2 = 6673177 + 27734375 = \boxed{34407552} = \boxed{34.4\times10^6\ cm^4}$$

$$I_{y_c} = \sum\left(I_{y_i} + A_i(dx_i)^2\right) = \sum I_{y_i} + \sum A_i(dx_i)^2 = 71907552 + 50000000 = \boxed{121907552} = \boxed{121.9\times10^6\ cm^4}$$

Question (2): (12 Marks)

Allowable **tensile** stress for concrete = 10 N/mm^2

Allowable **compressive** stress for steel = 140 N/mm^2



For Steel (Upper part):

$$\sigma_{Steel} = \frac{P_{Steel}}{A_{Steel}} = \frac{260\times10^3}{\pi(25)^2}$$

$$= 132.4\ N/mm^2 < 140\ N/mm^2$$ → **Safe** … (1)

For Steel (Lower part):

$$\sigma_{Steel} = \frac{P_{Steel}}{A_{Steel}} = \frac{390\times10^3}{\pi(30)^2}$$

$$= 137.9\ N/mm^2 < 140\ N/mm^2$$ → **Safe** … (2)

For Concrete:

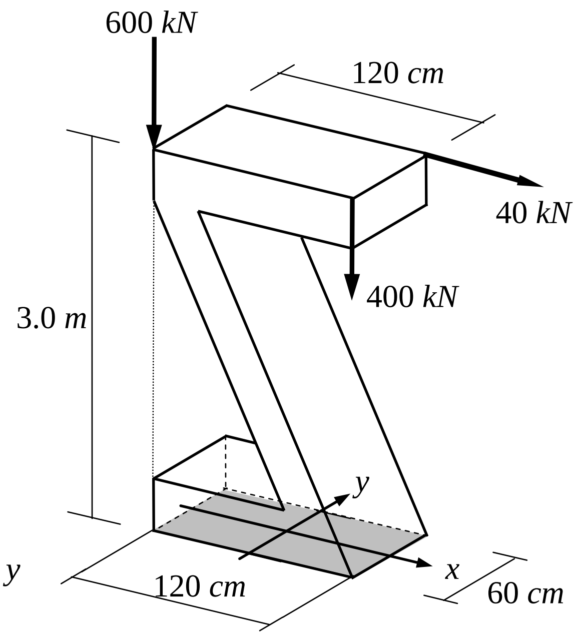
$$\sigma_{Con} = \frac{P_{Con}}{A_{Con}} = \frac{850\times10^3}{\pi(150)^2} = 12.0\ N/mm^2 > 10\ N/mm^2$$ → **Unsafe** … (3)

∴ From (1), (2) and (3), the column is **unsafe** to subject these axial loads.

With my best wishes
Dr. M. Abdel-Kader

Question (3): (12 Marks)

At the base section (120 *cm* × 60 *cm*) of the shown column, draw the normal stress distribution and calculate the maximum normal stresses. Neglect the column weight.



Solution:

N = - 600 - 400 = **- 1000 kN**

M_x = + 600×30 + 400×30 = **30000 kN.cm**

M_y = + 600×60 - 400×60 - 40×300 = **0**

A = 120×60 = **7200 cm²**

I_x = 120×60³/12 = **2160000 cm⁴**

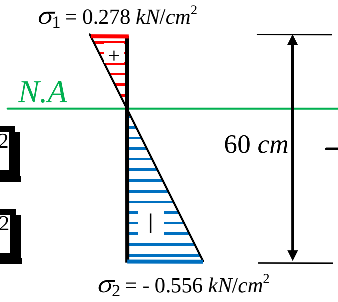
$$\sigma = \pm\frac{N}{A} \pm \frac{M_x}{I_x} y \pm \frac{M_y}{I_y} x = -\frac{1000}{7200} + \frac{30000}{2160000} y = -\frac{5}{36} + \frac{1}{72} y$$

N.A.

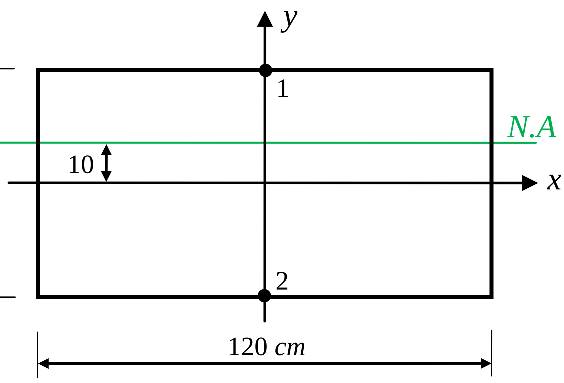
$$-\frac{5}{36} + \frac{1}{72} y = 0 \rightarrow y = 10 \text{ cm}$$

$$\sigma_1 = -\frac{5}{36} + \frac{1}{72}(30) = \boxed{+0.278 \text{ kN/cm}^2}$$

$$\sigma_2 = -\frac{5}{36} + \frac{1}{72}(-30) = \boxed{-0.556 \text{ kN/cm}^2}$$



Normal stress distribution



Question (4): (12 Marks)

(a) The twisting moment diagram is as shown

(b)

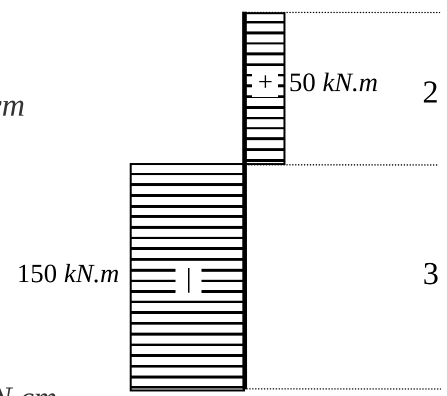
Part *AB*

Twisting moment = T = 50 *kN.m* = 5000 *kN.cm*

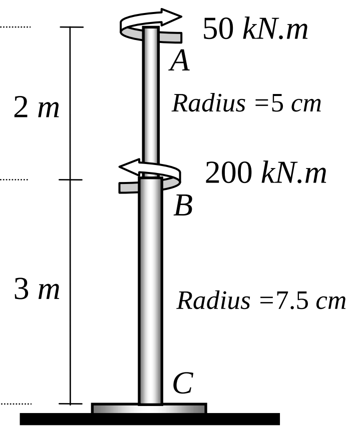
$J = \pi r^4/2 = \pi (5)^4/2 = 981.75 \text{ cm}^4$

So, $\tau = \frac{Tr}{J} = \frac{(5000)(5)}{981.75} = 25.46 \text{ kN/cm}^2$

$= \boxed{25.46 \text{ kN/cm}^2} = \boxed{254.6 \text{ MPa}}$



Twisting Moment Diagram



Part *BC*

Twisting moment = T = 150 *kN.m* = 15000 *kN.cm*

$J = \pi r^4/2 = \pi (7.5)^4/2 = 4970 \text{ cm}^4$

So, $\tau = \frac{Tr}{J} = \frac{(15000)(7.5)}{4970}$

$= 22.64 \text{ kN/cm}^2 = \boxed{22.64 \text{ kN/cm}^2} = \boxed{226.4 \text{ MPa}}$

(c)

$$\phi_{A/C} = \phi_{A/B} + \phi_{B/C} = \frac{5000 \times 200}{981.75 \times 8000} + \frac{-15000 \times 300}{4970 \times 8000} = 0.1273 - 0.1132 = 0.0141 \text{ rad} = \boxed{0.82^{\circ}}$$

With my best wishes

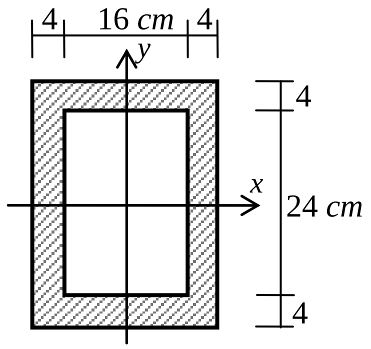
Dr. M. Abdel-Kader

Question (5): (12 Marks)

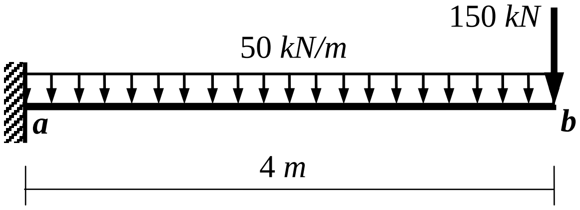
For the shown beam, calculate and draw:

(a) The **normal** stress distribution over the cross-section at ***a***.

(b) The **shear** stress distribution over the cross-section at ***a***.



Cross-section of the beam



Solution:

(a) Normal stress distribution over the cross-section at *b*.

Bending moment at a, $M = 150\times4+(50\times4)\times2 = 1000\ kN.m = 100000\ kN.cm$

$$I = \frac{24\times32^3}{12} - \frac{16\times24^3}{12} = 47104\ cm^4$$

$$\sigma_t = \sigma_c = \frac{M}{I}y = \frac{100000}{47104}(16) = 33.97\ kN/cm^2 = 339.7\ MPa$$

Maximum tensile normal stress = Maximum compressive normal stress ≅ **340 MPa**

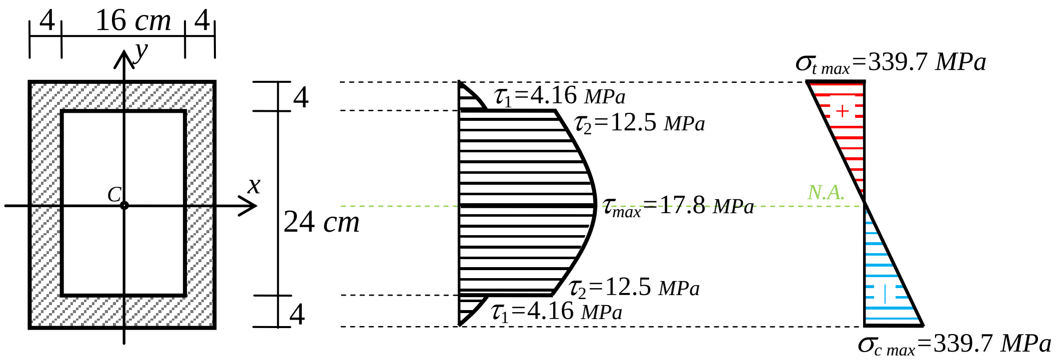
(b) Shear stress distribution over the cross-section at *a*.

Shear force at a, $Q = 150 + 50\times4 = 350\ kN = 350\times10^3\ N$

Shear stress, $\tau_1 = \frac{QS}{Ib} = \frac{(350\times10^3)(24\times4\times14)}{47104\times24} = 416\ N/cm^2$ $\quad$ **$\tau_1 = 4.16\ MPa$**

Shear stress, $\tau_2 = \frac{QS}{Ib} = \frac{(350\times10^3)(24\times4\times14)}{47104\times8} = 1248\ N/cm^2$ $\quad$ **$\tau_2 = 12.5\ MPa$**

Shear stress, $\tau_{max} = \frac{QS}{Ib} = \frac{(350\times10^3)(24\times4\times14+2\times12\times4\times6)}{47104\times8} = 1783\ kN/cm^2$ $\quad$ **$\tau_{max} = 17.8\ MPa$**



Cross-section of the beam

Shear stress distribution

Normal stress distribution

With my best wishes

Dr. M. Abdel-Kader