

Mid-Term Exam

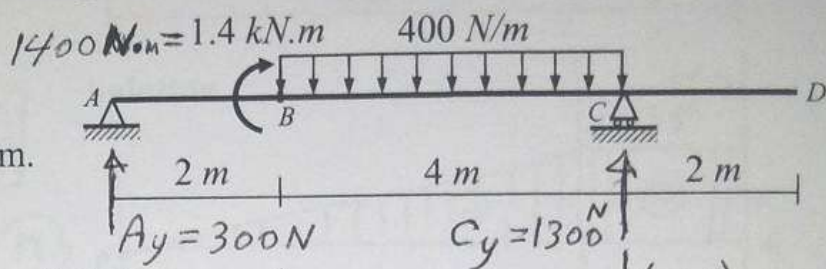
Question (1): (12 Marks)

For the shown beam, using the **double integration method**, determine:

- the deflection at B
- the deflection at D
- the slope at D

and sketch the elastic curve of the beam.

$$EI = 0.45 \times 10^6 \text{ N.m}^2$$



Reaction:

$$\sum M_c = 0$$

$$A_y(6) + 1400 - 1600(2) = 0$$

$$\therefore A_y = 300 \text{ N} \uparrow$$

$$\sum F_y = 0$$

$$300 - 1600 + C_y = 0$$

$$\therefore C_y = 1300 \text{ N} \uparrow$$

$$EI y'' = M$$

$$= 300x + 1400(x-2) - 400 \frac{(x-2)^2}{2} + 1300(x-6) + 400 \frac{(x-6)^2}{2}$$

$$EI y' = 150x^2 + 1400(x-2) - \frac{200}{3}(x-2)^3 + 650(x-6)^2 + \frac{200}{3}(x-6)^3 + C_1$$

$$EI y = 50x^3 + 700(x-2)^2 - \frac{50}{3}(x-2)^4 + \frac{650}{3}(x-6)^3 + \frac{50}{3}(x-6)^4 + C_1 x + C_2$$

B.C.: at $x=0$ $y=0 \Rightarrow C_2 = 0$

at $x=6$ $y=0 \Rightarrow 50(6)^3 + 700(4)^2 - \frac{50}{3}(4)^4 + \frac{650}{3}(2)^3 + \frac{50}{3}(2)^4 + C_1(6) = 0$

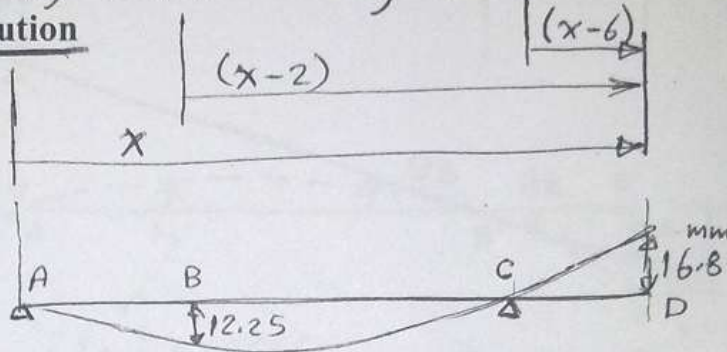
$$C_1 = -20600/9 = -2955.56$$

(a) $\delta_B = y_B = \frac{1}{EI} [50(2)^3 - 2955.56] = \frac{5511.1}{450000} = -0.012247 \text{ m}$

(b) $\delta_D = y_D = \frac{1}{EI} [50(8)^3 + 700(6)^2 - \frac{50}{3}(6)^4 + \frac{650}{3}(2)^3 + \frac{50}{3}(2)^4 - 2955.56(8)]$
 $= \frac{7555.56}{450000} = 0.01679 \text{ m}$

(c) $\theta_D = y'_D = \frac{1}{EI} [150(8)^2 + 1400(6) - \frac{200}{3}(6)^3 + 650(2)^2 + \frac{200}{3}(2)^3 - 2955.56]$
 $= 3777.78 / 450000 = 0.008395 \text{ rad}$

Solution



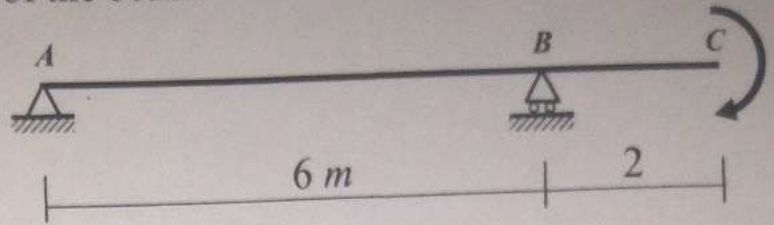
Elastic curve

Question (2): (8 Marks)

For the shown beam, using the **moment-area method**, determine the deflection at the right end **C** and sketch the elastic curve of the beam.

54 kN.m

$EI = 0.45 \times 10^6 \text{ N.m}^2$



$$t_{A/B} = \frac{1}{EI} \left[\text{Area}_{AB} \cdot \bar{X}_A \right]$$

$$= \frac{1}{EI} \left[\frac{1}{2}(6)(-54) \left(\frac{2}{3} \times 6 \right) \right]$$

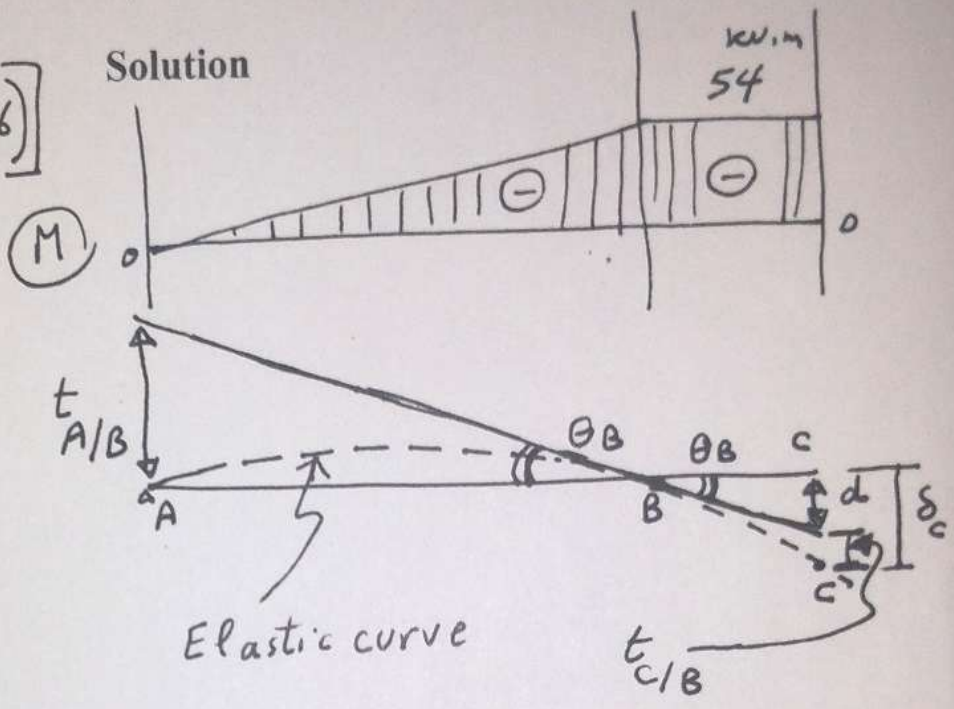
$$t_{A/B} = -\frac{648}{EI}$$

$$t_{C/B} = \frac{1}{EI} \left[\text{Area}_{BC} \cdot \bar{X}_C \right]$$

$$= \frac{1}{EI} \left[(2)(-54) \right]$$

$$t_{C/B} = -\frac{108}{EI}$$

Solution



$$\theta_B = \frac{t_{A/B}}{6} = \frac{d}{2} \implies d = \frac{t_{A/B}}{3}$$

$$\delta_c = t_{C/B} + d = -\frac{108}{EI} + -\frac{648}{3EI} = -\frac{324}{EI}$$

$$\delta_c = \frac{324}{450} = 0.72 \text{ m} \downarrow$$

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the deflection is very large
"EI" needs to be increased