Ministry of Higher Education
Giza Higher Institute for Eng. & Tech.
Civil Engineering Department
Course Name: **Theory of Structures (3)**
Course Code : **CIV 301**

Academic Year : **2014-2015**
Semester : **First**
Level : **3rd**
Time : **3 Hours**
Date : **10 / 1 / 2015**
Examiner: ***Dr. M. Abdel-Kader***

Solution of First Semester Final Exam

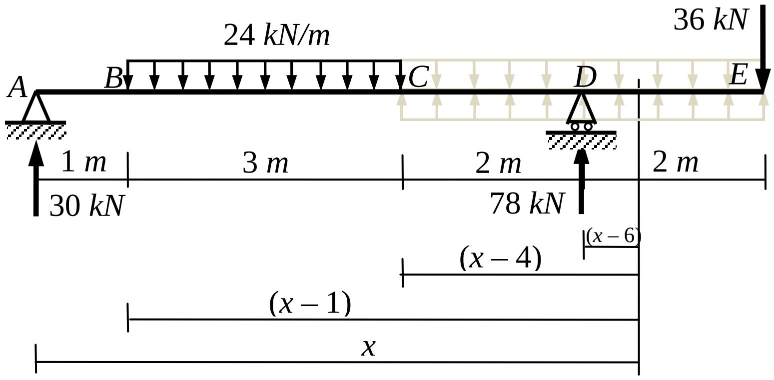
- Attempt all questions.
- The Exam consists of **5** questions in **1** page.
- Maximum grade is **60 Marks**

Question (1): (12 Marks)

For the shown beam, using the **double integration method**, determine:

(a) the deflections at *C* and *E*,
(b) determine the slope at *D*,
and sketch the elastic curve of the beam.

$EI = 10\times10^6\ N.m^2$



Reactions:

$A_y(6) + 36\times2 - 24\times3(3.5) = 0$

$\rightarrow A_y = 30\ kN$

$D_y + 30 - 24\times3 - 36 = 0$

$\rightarrow D_y = 78\ kN$

$M(x) = 30x - 24(x-1)^2/2 + 24(x-4)^2/2 + 78(x-6)$

$EIy'' = 30x - 12(x-1)^2 + 12(x-4)^2 + 78(x-6)$

$EIy' = 15x^2 - 4(x-1)^3 + 4(x-4)^3 + 39(x-6)^2 + C_1$

$EIy = 5x^3 - (x-1)^4 + (x-4)^4 + 13(x-6)^3 + C_1x + C_2$

Boundary Conditions:

At $x = 0$, $y = 0$ $\rightarrow$ $\underline{\mathbf{C_2 = 0}}$

At $x = 6\ m$, $y = 0$ $\rightarrow$ $0 = 5(6)^3 - (5)^4 + (2)^4 + 6C_1 +$ $\rightarrow$ $\underline{\mathbf{C_1 = -78.5}}$

So, the general equation of the deflection *y* at any distance *x* is,

$EIy = 5x^3 - (x-1)^4 + (x-4)^4 + 13(x-6)^3 - 78.5x$

(a) The deflection at *C* ($x = 4\ m$)

$EIy_C = 5(4)^3 - (3)^4 - 78.5(4) = -75$

$y_C = -75/10000 = 0.0075\ m = -7.5\ mm$ **$y_C = 7.5\ mm$ ⬇**

The deflection at *E* ($x = 8\ m$)

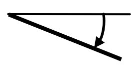
$EIy_E = 5(8)^3 - (7)^4 + (4)^4 + 13(2)^3 - 78.5(8) = -109$

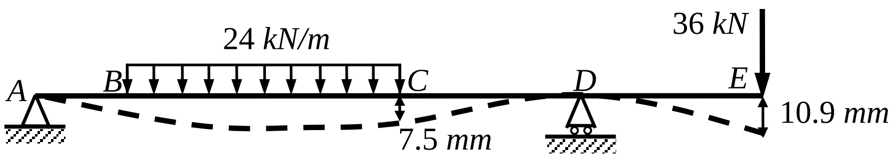
$y_E = -109/10000 = 0.0109\ m = -10.9\ mm$ **$y_E = 10.9\ mm$ ⬇**

(b) The slop at *D* ($x = 6\ m$)

$EIy'_D = 15(6)^2 - 4(5)^3 + 4(2)^3 - 78.5 = -6.5$

$y'_D = -6.5/10000 = -0.00065 = -0.03724^o$ **$y'_D = -0.037^o$**





Elastic Curve

Question (2): (12 Marks)

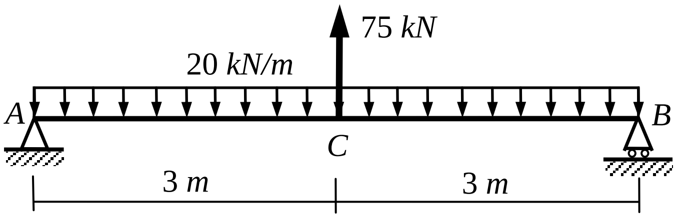
For the shown beam, using the **moment-area method**, determine:

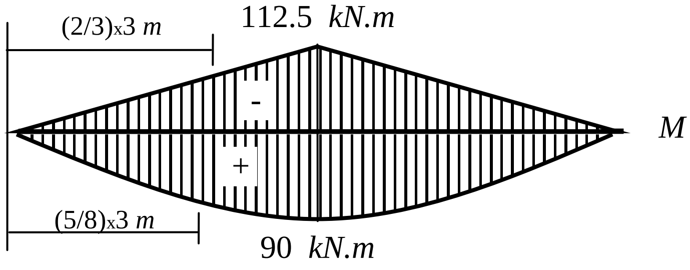
(a) the slope at ***A***,

(b) the deflection at ***C***,

and sketch the elastic curve of the beam.

$EI = 20\times10^{6}\ N.m^{2}$





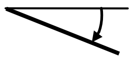
(a) Since the slope at ***C*** (θ_C) is equal to zero, the change in slope between the tangents of the elastic curve at points *A* and *B* (θ_{AC}) is equal to the slope at ***A*** (θ_A),

$$\theta_{AC} = \theta_A - \theta_C = \theta_A - 0 = \theta_A$$

then

$$\theta_A = \frac{1}{EI}\left[Area_{AC}\right] = \frac{1}{20000}\left[\tfrac{2}{3}\times3\times90 - \tfrac{1}{2}\times3\times112.5\right] = \frac{11.25}{20000} = 5.625\times10^{-4}\ rads$$

$$= 0.0322\ degrees \qquad \therefore \boxed{\theta_A = 0.0322^{o}}$$

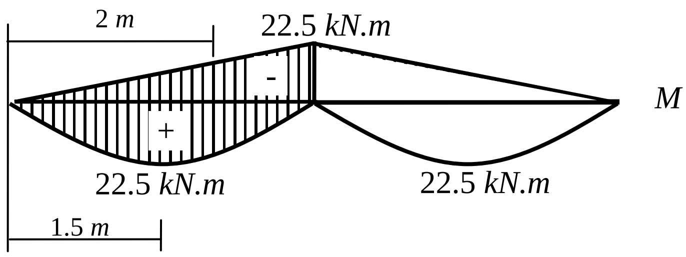


(b) The deflection at ***C*** (δ_C) is equal to the deviation of the point ***A*** above the tangent to the elastic curve at ***C***, then

$$\delta_C = t_{A/C} = \frac{1}{EI}\left[Area_{AC}.\bar{X}_A\right]$$

$$= \frac{1}{20000}\left[(\tfrac{2}{3}\times3\times90)(\tfrac{5}{8}\times3) - (\tfrac{1}{2}\times3\times112.5)(\tfrac{2}{3}\times3)\right] = 0 \quad \therefore \boxed{\delta_C = 0}$$

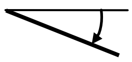
OR



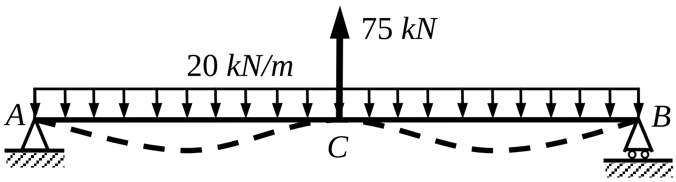
(a) $$\theta_A = \frac{1}{EI}\left[Area_{AC}\right]$$

$$= \frac{1}{20000}\left[\tfrac{2}{3}\times3\times22.5 - \tfrac{1}{2}\times3\times22.5\right] = \frac{11.25}{20000} = 5.625\times10^{-4}\ rads$$

$$= 0.0322\ degrees \qquad \therefore \boxed{\theta_A = 0.0322^{o}}$$



(b) $$\delta_C = t_{A/C} = \frac{1}{20000}\left[(\tfrac{2}{3}\times3\times22.5)(\tfrac{1}{2}\times3) - (\tfrac{1}{2}\times3\times22.5)(\tfrac{2}{3}\times3)\right] = 0 \quad \therefore \boxed{\delta_C = 0}$$



Elastic Curve

Question (3): **(12 Marks)**

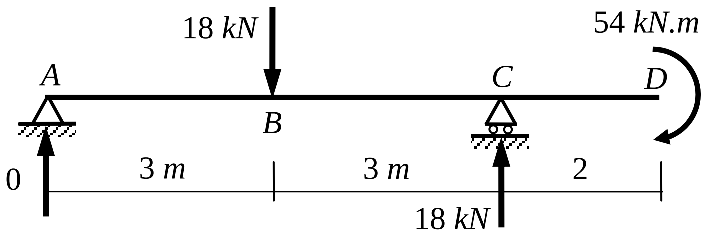
For the shown beam, using the **conjugate beam method**, determine:

(a) the slope at ***C***.

(b) the deflections at *B* and ***D***.

and sketch the elastic curve of the beam.

$EI = 20\times10^3\ kN.m^2$

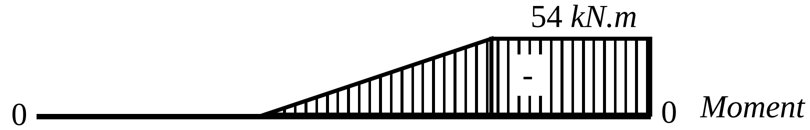


Reaction:

$+\circlearrowright \sum M_A = 0$

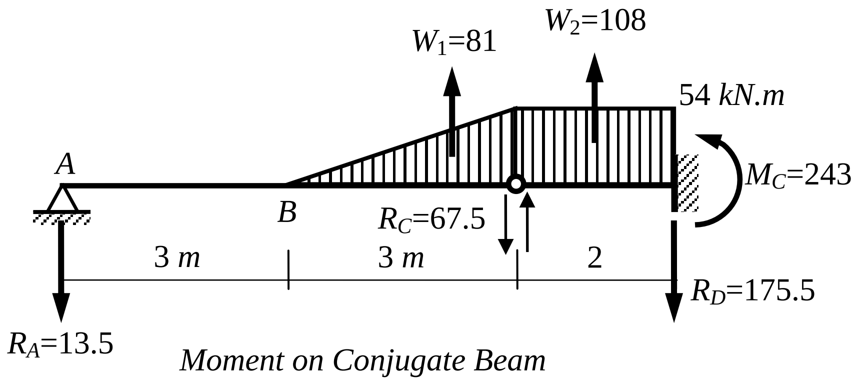
$A_y(6) - 18\times3 + 54 = 0 \rightarrow A_y = 0$

First construct the bending moment diagram of the real beam.



The resulting moment diagram is then loaded to the conjugate beam.

For the conjugate beam, determine the elastic reaction (R_C, M_B and M_D).



Moment on Conjugate Beam

$W_1 = \frac{1}{2}\times3\times54 = 81\ \ kN.m^2$

$W_2 = 2\times54 = 108\ kN.m^2$

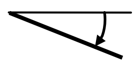
M_C for left part $= 0$

$R_A(6) - W_1(1) = 0 \rightarrow R_A = 13.5\ kN.m^2 \quad \rightarrow R_C = 81 - 13.5 = 67.5\ kN.m^2$

$M_B = -R_A(3) = -13.5(3) = -40.5\ kN.m^3$

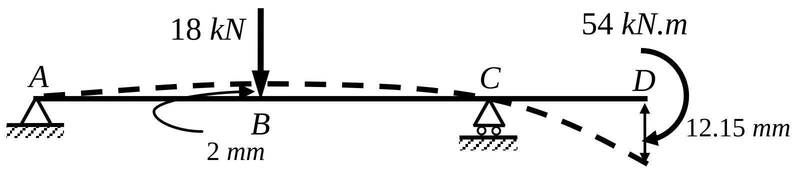
$M_D = R_C(2) + W_2(1) = 67.5(2) + 108(1) = 243\ kN.m^3$

(a) Slope at C = $R_C / EI = 67.5 / 20000 = 0.003375$ rad $= 0.1934^o$ ∴ **$\theta_c = 0.19^o$**



(b) Deflection at $B = M_B / EI = -40.5 / 20000 = -0.002025\ m = -2\ mm$ ∴ **$\delta_B = 2\ mm$ ⬆**

Deflection at $D = M_D / EI = 243 / 20000 = 0.01215\ m = 12.15\ mm$ ∴ **$\delta_D = 12.15\ mm$ ⬇**

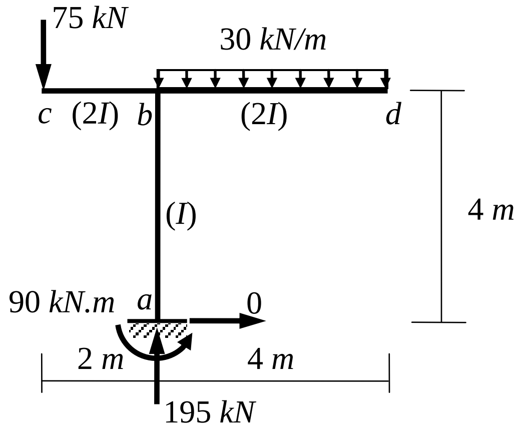


Elastic Curve

Question (4): (12 Marks)

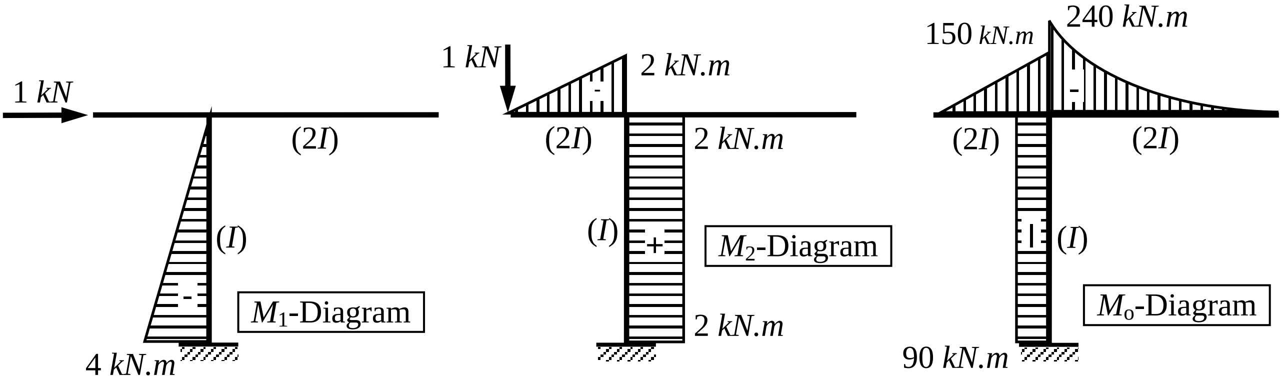
For the shown frame and truss, using the **virtual work method**, determine the horizontal and vertical displacements at *c* (δ_{ch} and δ_{cv}).

For the frame, the relative moments of inertia are given between brackets and $EI = 20\times10^3$ $kN.m^2$. For the truss, assume that all members have the same axial rigidity $EA = 1000$ *kN*.

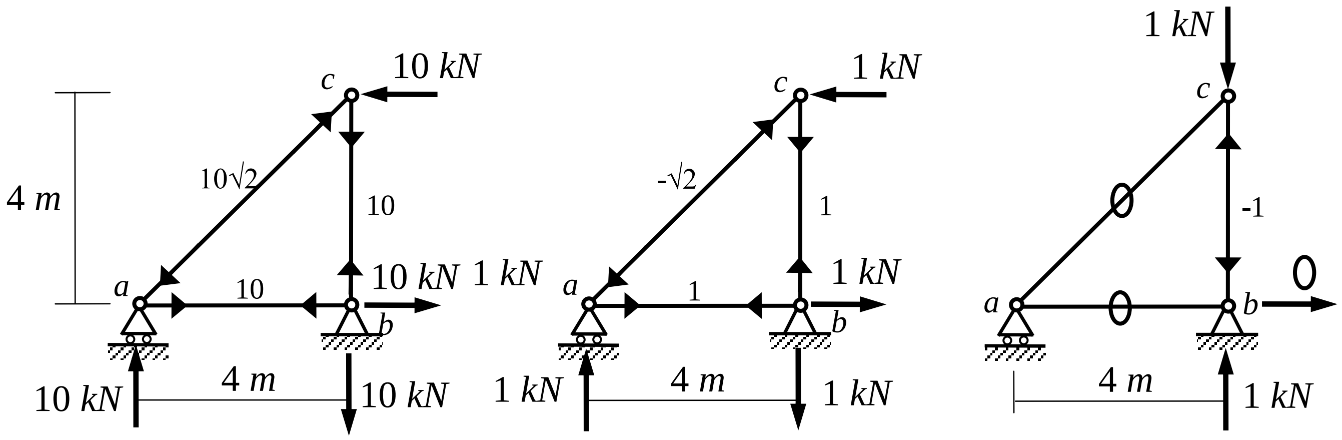


$$\delta_{ch} = \int \frac{M_o M_1}{EI} dL = \frac{1}{EI}\left[(-4\times90)(-\tfrac{1}{2}\times4)\right] = \frac{720}{EI}$$

$$\therefore \boxed{\delta_{ch} = \frac{720}{20000} = 0.036\ m = 36\ mm \rightarrow}$$



$$\delta_{cv} = \int \frac{M_o M_2}{EI} dL = \frac{1}{EI}\left[(-4\times90)(2)\right] + \frac{1}{2EI}\left[(-\tfrac{1}{2}\times2\times150)(-\tfrac{2}{3}\times2)\right] = \frac{-620}{EI} \therefore \boxed{\delta_{cv} = -\frac{620}{20000} = 0.031\ m = 31\ mm \uparrow}$$



(N$_o$) (N$_1$) (N$_2$)

Reactions and member forces due to:

(a) Applied load (b) Horizontal unit load at *c* (c) Vertical unit load at *c*

Calculation details for horizontal and vertical deflection of joint *c*

Member	L (*m*)	EA (*kN*)	N_o (*kN*)	N_1 (*kN*)	N_2 (*kN*)	N_oN_1L / EA	N_oN_2L / EA
ab	4	1000	10	1	0	0.04	0
bc	4	1000	10	1	-1	0.04	-0.04
ac	5.657	1000	-14.14	-1.414	0	0.113	0
$\sum$						δ_{ch}=0.193 *m* (= 19.3 *mm* left)	δ_{cv}= -0.04 *m* (= 4 *mm* Upwards)

Horizontal deflection of joint c = $\delta_c^h = \sum \frac{N_o N_1 L}{EA} = 193\ mm \leftarrow$

Vertical deflection of joint c = $\delta_c^v = \sum \frac{N_o N_2 L}{EA} = 40\ mm \uparrow$

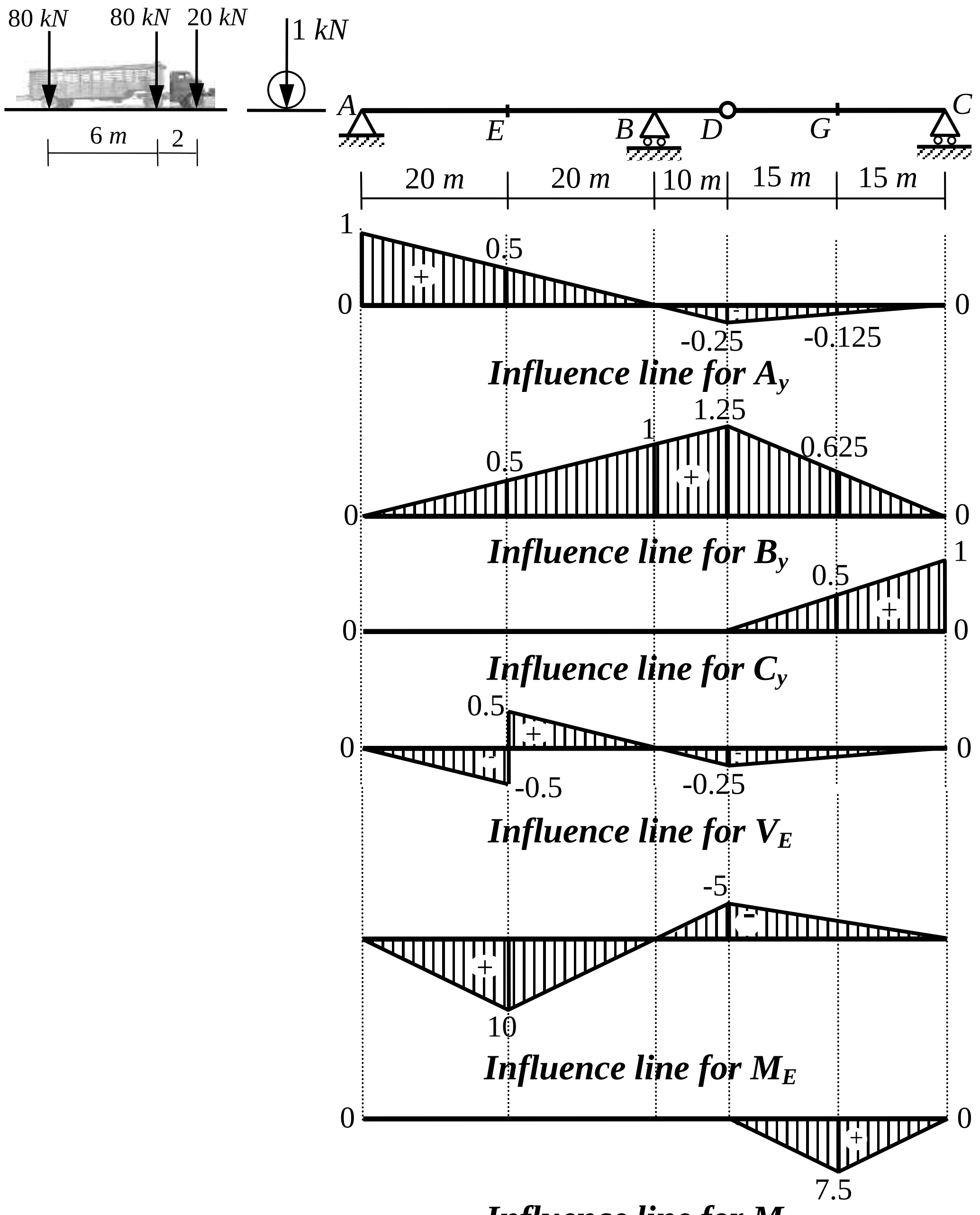
Question (5): **(12 Marks)**

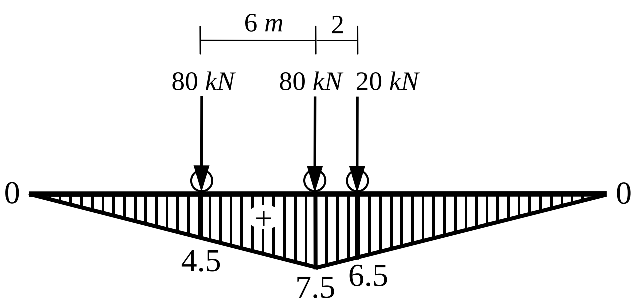
For the shown beam, draw the influence line for:

(a) the reactions A_y, B_y and C_y.

(b) the shear force at the section E and the bending moments at the sections E and G.

Also, determine the maximum moment at G caused by the shown moving truck.





I. L. for M_G

$$M_{G\,max} = 80(4.5) + 80(7.5) + 20(6.5)$$
$$= 1090\ kN.m$$

$\therefore$ $M_{G\ max} = 1090\ kN.m$

With my best wishes

Dr. M. Abdel-Kader