Ministry of Higher Education
Giza Higher Institute of Engineering & Technology
Civil Engineering Department
Course Name: **Theory of Structures (1)B**
Course Code : **CIV 121** Date: **19 / 5 / 2019**

Academic Year : **2018/2019**
Semester : **Second**
Level : **1st Civil**
Time : 3 **Hours**
Examiner**:** ***Dr. M. Abdel-Kader***

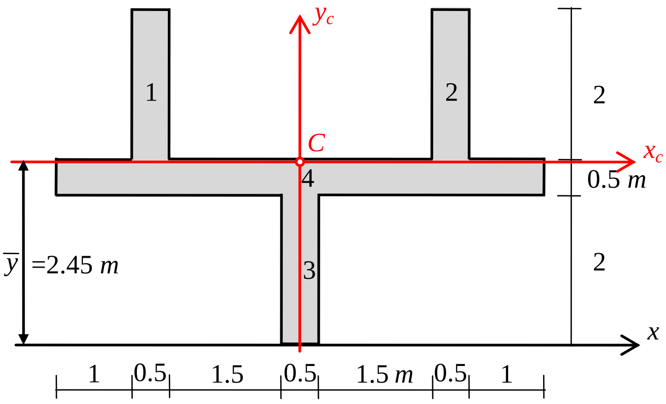
Answer of Final Exam

Total Marks: **90** No. of Questions**: 3** (Attempt all questions)

Question (1): (15 Marks)

(a) For the shown cross-section, determine the following:
- The location of the centroid.
- The moments of inertia about the centroidal axes.
- The polar moment of inertia.
- The radii of gyration (r_x & r_y).

Note: *Divide the cross-section to 4 elements as shown on the figure.*



Element	A_i	y_i	$A_i\,y_i$	I_{xi}	$dy_i=y_i-\bar{y}$	$A_i\,(dy_i)^2$	I_{yi}	dx_i	$A_i\,(dx_i)^2$
1	0.5×2	3.5	3.5	1/3	1.05	1.1025	1/48	-2	4
2	0.5×2	3.5	3.5	1/3	1.05	1.1025	1/48	2	4
3	0.5×2	1	1	1/3	-1.45	2.1025	1/48	0	0
4	0.5×6.5	2.25	7.3125	13/192	-0.2	0.13	2197/192	0	0
Σ	6.25	-	15.3125	205/192=1.0677	-	4.4375	2209/192=11.5052		8

(a) Location of the centroid:

- From symmetry y_c-axis is as shown
- $\bar{y} = \dfrac{\sum A_i\, y_i}{\sum A_i} = \dfrac{15.3125}{6.25} = $ **2.45 *m***

(b) The moments of inertia about the centroidal axes

$$I_{x_c} = \sum\left((I_{x_i} + A_i(dy_i)^2\right) = \sum I_{x_i} + \sum A_i(dy_i)^2 = 1.0677 + 4.4375 = 5.5052 = \boxed{5.5\ m^4}$$

$$I_{y_c} = \sum\left((I_{y_i} + A_i(dx_i)^2\right) = \sum I_{y_i} + \sum A_i(dx_i)^2 = 11.5052 + 8 = 19.5052 = \boxed{19.5\ m^4}$$

(c) The polar moment of inertia (I_p) = $I_{xc}+I_{yc}$ = 5.5 + 19.5 = $\boxed{25.0\ m^4}$

(d) $r_x = \sqrt{I_x / A} = \sqrt{5.5/6.25} = \boxed{0.938\ m}$ and $r_y = \sqrt{I_y / A} = \sqrt{19.5/6.25} = \boxed{1.766\ m}$

With my best wishes
Dr. M. Abdel-Kader

Question (1): (15 Marks)

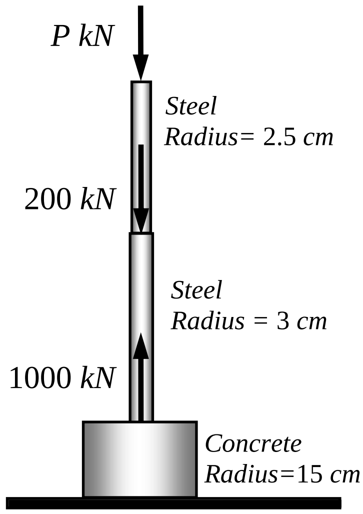
(b) A column of variable circular cross-section is subjected to axial loads as shown. Determine the safe range of ***P***.

Given Data:

For **Steel:** Allowable compressive and tensile stresses = 140 *MPa*

For **Concrete:** Allowable compressive stress = 80 *MPa*

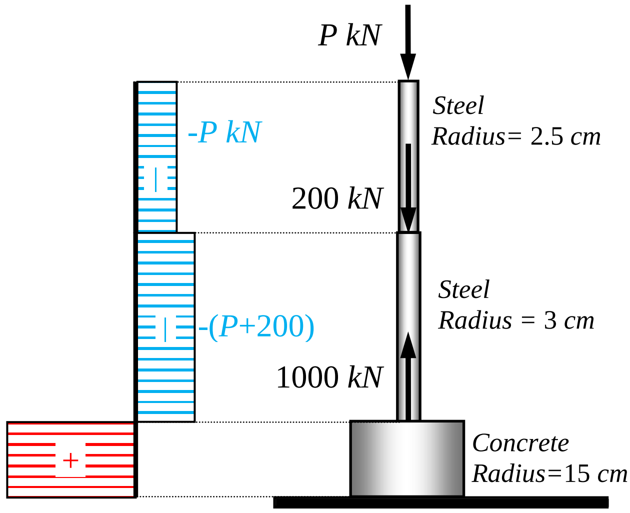
Allowable tensile stress = 10 *MPa*



Solution:

Allowable **tensile** stress for concrete = 10 N/mm^2

Allowable **compressive** stress for steel = 140 N/mm^2



N.F.D

For Steel (Upper part):

$$\sigma_{Steel} = \frac{P_{Steel}}{A_{Steel}} = \frac{P \times 10^3}{\pi (25)^2} \leq 140\ N/mm^2$$

→ $P \leq 140 \times \pi (25)^2 / 1000$

→ $P \leq 274.89\ kN$ … (1)

For Steel (Lower part):

$$\sigma_{Steel} = \frac{P_{Steel}}{A_{Steel}} = \frac{(P+200) \times 10^3}{\pi (30)^2} \leq 140\ N/mm^2$$

→ $P+200 \leq 140 \times \pi (30)^2 / 1000$

→ $P+200 \leq 395.841$

→ $P \leq 195.84\ kN$ … (2)

For Concrete:

$$\sigma_{Con} = \frac{P_{Con}}{A_{Con}} = \frac{(800-P) \times 10^3}{\pi (150)^2} \leq 10\ N/mm^2$$

→ $800 - P \leq 10 \times \pi (150)^2 / 1000$

→ $800 - P \leq 706.86$

→ $-P \leq -93.14$

→ $P \geq 93.14\ kN$ … (3)

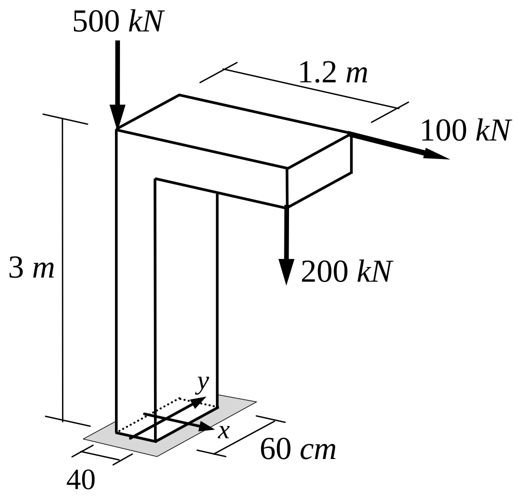
∴ From (1), (2) and (3), the safe range of axial force ***P*** is (93.14 *kN* to 195.84 *kN*)

With my best wishes

Dr. M. Abdel-Kader

Question (2): (15 Marks)

(a) For the shown column, draw the normal stress distribution at the base section (40 *cm* × 60 *cm*) and calculate the maximum normal stresses. Neglect the column weight.



Solution:

$N = -500 - 200 = -700\ kN$

$M_x = +500\times30 + 200\times30 = 21000\ kN.cm$

$M_y = +500\times20 - 200\times100 - 100\times300 = -40000\ kN.cm$

$A = 40\times60 = 2400\ cm^2$

$I_x = 40\times60^3/12 = 720000\ cm^4$

$I_y = 60\times40^3/12 = 320000\ cm^4$

$$\sigma = \pm\frac{N}{A} \pm \frac{M_x}{I_x}y \pm \frac{M_y}{I_y}x = -\frac{700}{2400} + \frac{21000}{720000}y - \frac{40000}{320000}x$$

N.A.

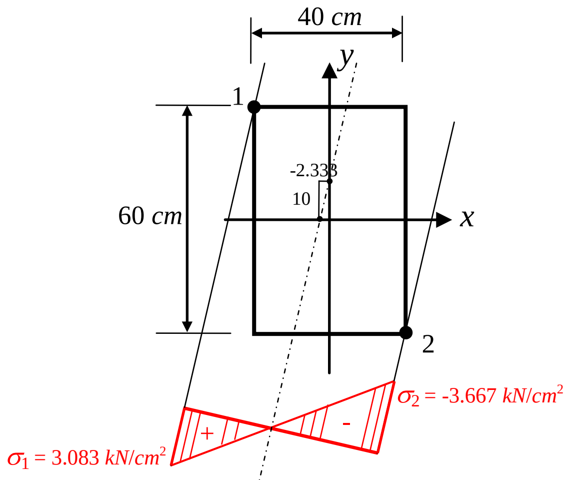
$$-\frac{7}{24} + \frac{7}{240}y - \frac{1}{8}x = 0$$

At $x = 0$; $y = +\frac{7\times240}{24\times7} = 10\ cm$ → (0, 10)

At $y = 0$; $x = -\frac{7\times8}{24\times1} = -7/3 = -2.333\ cm$ → (-2.333, 0)

$$\sigma_1 = -\frac{7}{24} + \frac{7}{240}(30) - \frac{1}{8}(-20) = 37/12 = \boxed{+3.083\ kN/cm^2}$$

$$\sigma_2 = -\frac{7}{24} + \frac{7}{240}(-30) - \frac{1}{8}(20) = -11/3 = \boxed{-3.667\ kN/cm^2}$$

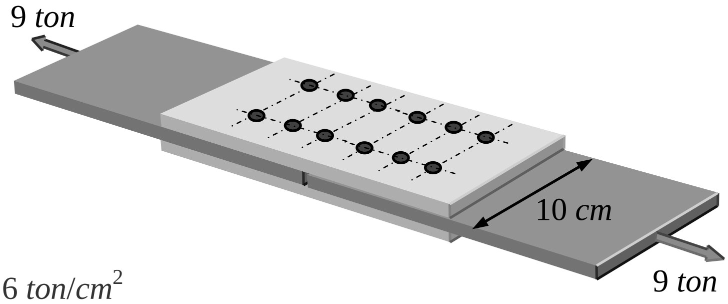


Normal stress distribution

With my best wishes

Dr. M. Abdel-Kader

Question (2): (15 Marks)



(b) For the shown bolted butt joint, determine the safe diameter of bolts. The width of plates is 10 *cm*, and the thickness of plates is 12 *mm*. The allowable stresses are as follows:

Bolts: $\tau_{all} = 1.1\ ton/cm^2$,

Plates: $\sigma_{t\ all} = 1.4\ ton/cm^2$ and $\sigma_{bearing\ all} = 1.6\ ton/cm^2$

Solution:

1. Failure due to bolt shear:

$$\tau_{all} \geq \frac{Q}{n\left(\pi d^2/4\right)} \rightarrow 1.1 \geq \frac{9}{(6\times 2)\left(\pi d^2/4\right)} \rightarrow d \geq 0.932\ cm \qquad \text{------- (1)}$$

2. Bearing failure of bolt or plate (compression failure):

$$\sigma_{bearing\ all} \geq \frac{Q}{n(dt)} \rightarrow 1.6 \geq \frac{9}{6(d\times 1.2)} \rightarrow d \geq 0.781\ cm \qquad \text{------- (2)}$$

3. Plate tearing:

$$\sigma_{t\ all} \geq \frac{Q}{(w-n\,d)\,t} \rightarrow 1.4 \geq \frac{9}{(10-2d)\times 1.2} \rightarrow 10-2d \geq \frac{9}{1.4\times 1.2}$$

$$\rightarrow -2d \geq \frac{9}{1.4\times 1.2} - 10 \rightarrow -2d \geq -4.64 \rightarrow d \leq 2.32\ cm \qquad \text{------- (3)}$$

From (1)-(3), the safe diameter of bolts is ≥ 0.932 *cm* and ≤ 2.32 *cm*

the safe diameter of bolts is 0.932 *cm* = 9.32 *mm* (take *d* =10 *mm*)

Check

$$\tau_{all} = \frac{Q}{n\left(\pi d^2/4\right)} = \frac{9}{12\left(\pi 1^2/4\right)} = 0.955 < 1.1\ ton/cm^2 \ \text{ O.K.}$$

$$\sigma_{bearing\ all} = \frac{Q}{n(dt)} = \frac{9}{6(1\times 1.2)} = 1.25 < 1.6\ ton/cm^2 \ \text{ O.K.}$$

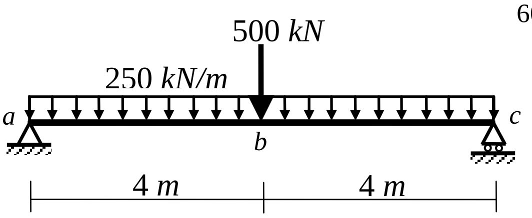
$$\sigma_{t\ all} = \frac{Q}{(w-n\,d)\,t} = \frac{9}{(10-2\times 1)\times 1.2} = 0.94 < 1.4\ ton/cm^2 \ \text{ O.K.}$$

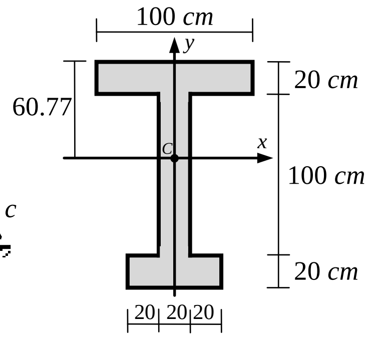
With my best wishes

Dr. M. Abdel-Kader

Question (3): (15 Marks)

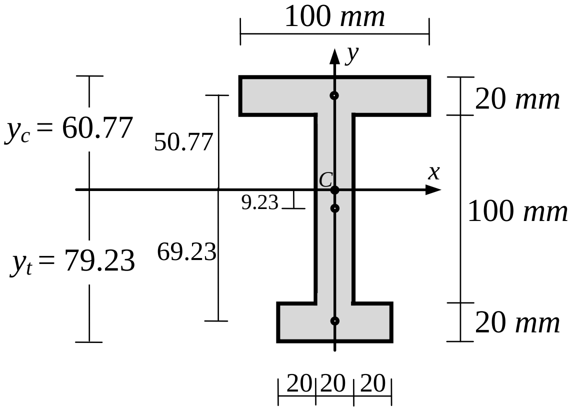
(a) For the shown beam, calculate and draw the shear stress distribution over the cross-section at ***a***.

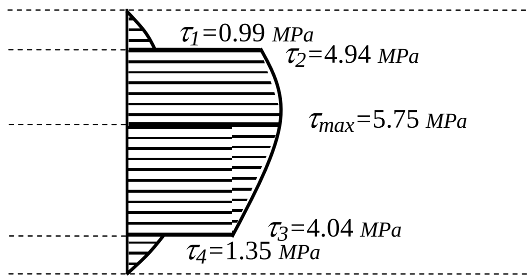




Cross-section of the beam

Solution:





Shear stress distribution

$I_x = [60\times20^3/12 + (1200)(-69.23)^2] + [20\times100^3/12 + (2000)(-9.23)^2] + [100\times20^3/12 + (2000)(50.77)^2]$
$= 12850256\ cm^4 = 12.85\times10^6\ cm^4$

Shear force at a, $Q_a = wL/2 + P/2 = 250\times8/2 + 500/2 = 1000+250 = 1250\ kN = 1.25\times10^6\ N$

Shear stress, $\tau_1 = (Q_a S_x) / (I_x b) = (1.25\times10^6 \times 2000\times50.77) / (12.85\times10^6\times100)$
$= 98.77\ N/cm^2 = 0.99\ MPa$ **$\tau_1 = 0.99\ MPa$**

Shear stress, $\tau_2 = (1.25\times10^6 \times 2000\times50.77) / (12.85\times10^6\times20)$
$= 493.87\ N/cm^2 = 4.94\ MPa$ **$\tau_2 = 4.94\ MPa$**

Shear stress, $\tau_{max} = (1.25\times10^6)(2000\times50.77+20\times40.77^2/2) / (12.85\times10^6\times20)$
$= 574.72\ N/cm^2 = 5.75\ MPa$ **$\tau_{max} = 5.75\ MPa$**

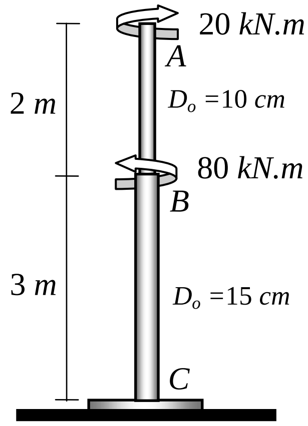
Shear stress, $\tau_3 = (1.25\times10^6)(1200\times69.23) / (12.85\times10^6\times20)$
$= 404.1\ N/cm^2 = 4.04\ MPa$ **$\tau_3 = 4.04\ MPa$**

Shear stress, $\tau_4 = (1.25\times10^6)(1200\times69.23) / (12.85\times10^6\times60)$
$= 134.689\ N/cm^2 = 1.35\ MPa$ **$\tau_4 = 1.35\ MPa$**

With my best wishes

Dr. M. Abdel-Kader

Question (3): (15 Marks)

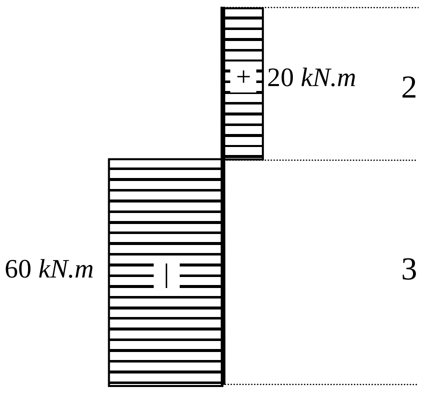


(b) A column of two tubes must resist torques as shown. The tubes have outside diameters of 10 *cm* and 15 *cm* and a thickness of 10 *mm*.

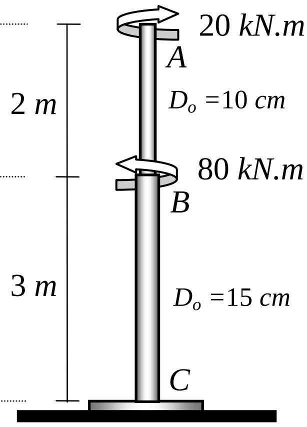
- Draw the twisting moment diagram.
- Determine the maximum shear stress τ_{max} in the two tubes.
- Determine the relative angle of twist ϕ between ***A*** and ***C***, where G = 30 *GPa*

Solution:

The twisting moment diagram is as shown



Twisting Moment Diagram



- The twisting moment diagram is as shown.
-

Part *AB*

Twisting moment, $T = 20\ kN.m = 20\times10^6\ N.mm$

$$J = \frac{\pi(r_o^4 - r_i^4)}{2} = \frac{\pi(50^4 - 40^4)}{2} = 5.796\times10^6\ mm^4$$

$$\text{So, } \tau_{max} = \frac{Tr_o}{J} = \frac{(20\times10^6)(50)}{5.796\times10^6} = 172.5\ N/mm^2$$

$$= \boxed{172.5\ MPa}$$

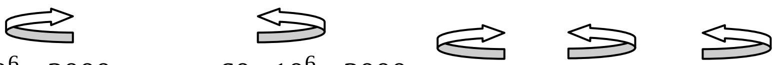
Part *BC*

Twisting moment, $T = 60\ kN.m = 60\times10^6\ N.mm$

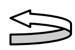
$$J = \frac{\pi(r_o^4 - r_i^4)}{2} = \frac{\pi(75^4 - 65^4)}{2} = 21.661\times10^6\ mm^4$$

$$\text{So, } \tau_{max} = \frac{Tr_o}{J} = \frac{(60\times10^6)(75)}{21.661\times10^6} = 207.7\ N/mm^2 = \boxed{207.7\ MPa}$$

-



$$\phi_{A/C} = \phi_{A/B} + \phi_{B/C} = \frac{20\times10^6\times2000}{5.796\times10^6\times30000} + \frac{-60\times10^6\times3000}{21.661\times10^6\times30000} = 0.23 - 0.277 = 0.047 \text{ rad} = \boxed{2.7^o}$$



With my best wishes

Dr. M. Abdel-Kader