Ministry of Higher Education
Giza Higher Institute of Engineering & Technology
Civil Engineering Department
Course Name: **Theory of Structures (2)A**
Course Code : **CIV 211** Date : **11 / 11 / 2019**

Academic Year : **2019/2020**
Semester : **First**
Level : **2 Civil**
Time : 1 **Hour**
Examiner**:** ***Dr. M. Abdel-Kader***

Answer of Mid-Term Exam

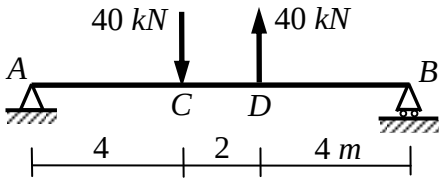
Total Marks**: 15** No. of Questions**:1**

Name: **Code:**

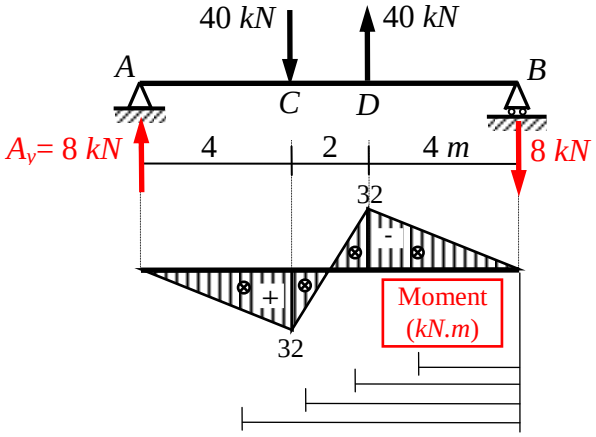
Question (1): (15 Marks)

a) Using the **moment-area method**, determine the slope at ***A*** and the deflection at ***D***.
b) Using the **conjugate beam method**, determine the slope at ***B*** and the deflection at ***C***.
c) Sketch the elastic curve of the beam.

$EI = 1\times10^4\ kN.m^2$



Solution:



a) Reaction:

$+\circlearrowleft \sum M_B = 0$

$A_y\,(10) - 40\times6 + 40\times4 = 0 \rightarrow A_y = 8\ kN\uparrow$

$B_y = 8\ kN\downarrow$

The bending moment diagram is as shown.

The slope at A (θ_A)

$$\theta_A = \frac{t_{B/A}}{10}$$

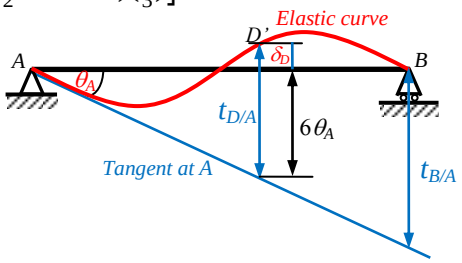
$$t_{B/A} = \frac{1}{EI}\left[Area_{AB}.\bar{X}_B\right]$$

$$= \frac{1}{EI}\left[(\tfrac{1}{2}\times4\times32)(6+\tfrac{4}{3}) + (\tfrac{1}{2}\times1\times32)(5+\tfrac{2}{3}) + (-\tfrac{1}{2}\times1\times32)(4+\tfrac{1}{3}) + (-\tfrac{1}{2}\times4\times32)(\tfrac{8}{3})\right]$$

$$= \frac{320}{EI} = \frac{320}{10000} = 0.032\ m$$

$$\therefore \theta_A = \frac{t_{B/A}}{10} = \frac{0.032}{10} = 0.0032\ rad = 0.1833^o$$

$\theta_A = 0.1833^o\ \circlearrowleft$



The deflection at D (δ_D)

$\delta_D = t_{D/A} - 6\,\theta_A$

Applying the second moment-area theorem, then

$$t_{D/A} = \frac{1}{EI}\left[\text{First moment of area of M - diagram between } A \text{ and } D \text{ about } D\right]$$

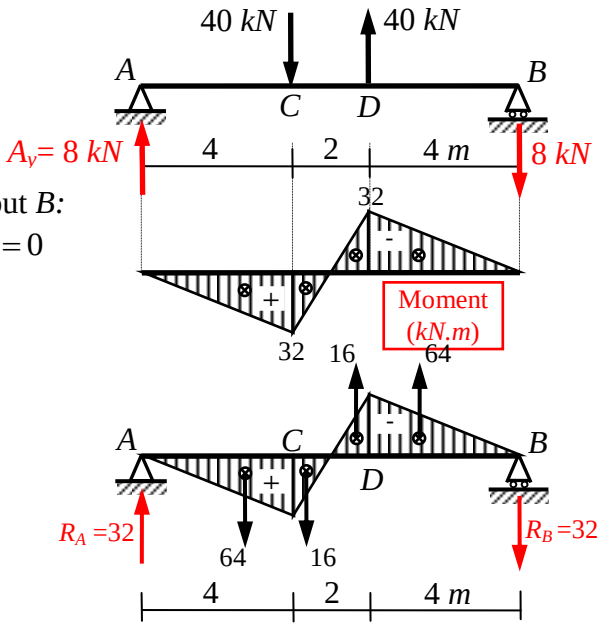
$$= \frac{1}{EI}\left[Area_{AD}.\bar{X}_D\right] = \frac{1}{EI}\left[(\tfrac{1}{2}\times4\times32)(2+\tfrac{4}{3}) + (\tfrac{1}{2}\times1\times32)(1+\tfrac{2}{3}) + (-\tfrac{1}{2}\times1\times32)(\tfrac{1}{3})\right] = \frac{704}{3EI} = \frac{704}{3\times10000}$$

$= 0.023467\ m$

$\therefore \delta_D = 0.023467 - 6(.0032) = 0.004267\ m$ $\quad \delta_D = 4.27\ mm\uparrow$

With my best wishes
Dr. M. Abdel-Kader

b) The bending moment diagram is as shown.
The resulting moment diagram is then loaded to the conjugate beam.
For the conjugate beam, determine the elastic reaction (r_A and r_B).



To determine the elastic reaction at A (R_A) take moment about B:

$$R_A(10) - (64)(6+\tfrac{4}{3}) - (16)(5+\tfrac{2}{3}) + (16)(4+\tfrac{1}{3}) + (64)(\tfrac{8}{3}) = 0$$

$\rightarrow R_A = 32 \uparrow$ $\rightarrow$ $R_B = 32 \downarrow$

Slope at B = R_B / EI = 32 / 10000 = 0.0032 rad = 0.1833°

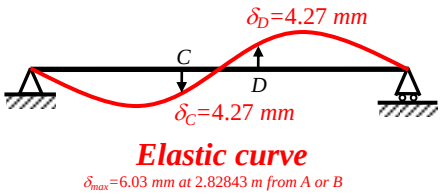
$\theta_A = 0.1833^o \circlearrowright$

Deflection at C = M_C / EI = [32(4) – 64(4/3)] / 10000

δ_C = 128 / 30000 = 0.004267 m

$\therefore \delta_C = 4.27\ mm \downarrow$

c)



Elastic curve

δ_{max}=6.03 *mm at* 2.82843 *m from A or B*

With my best wishes

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