Ministry of Higher Education
Giza Higher Institute of Engineering & Technology
Civil Engineering Department
Course Name: **Theory of Structures (2)A**
Course Code : **CIV 211** Date : **29 / 12 / 2019**

Academic Year : **2019/2020**
Semester : **First**
Level : **2 Civil**
Time : 3 **Hours**
Examiner: ***Dr. M. Abdel-Kader***

Answer of Final Exam

Total Marks: **70** No. of Questions: **5**

Question (1): (14 Marks)

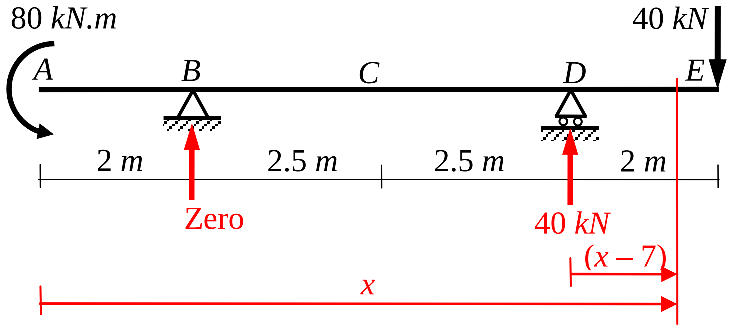
For the shown beam, using the **double integration method**, determine:

(a) the deflections at ***A***, ***C*** and ***E***

(b) the slopes at ***A*** and ***C***

and sketch the elastic curve of the beam.

$EI = 5\times10^4\ kN.m^2$



Reactions:

$\circlearrowleft M_D = -80 + B_y(5) + 40\times2 = 0 \rightarrow B_y = 0$

$\uparrow F_y = D_y - 40 = 0 \rightarrow D_y = 40\ kN \uparrow$

$$M = -80\,(x)^0 + 40\,(x-7)$$
$$EIy'' = -80\,(x)^0 + 40\,(x-7)$$
$$EIy' = -80\,x + 20\,(x-7)^2 + C_1$$
$$EIy = -40\,x^2 + 20\,(x-7)^3/3 + C_1x + C_2$$

Boundary Conditions:

At $x = 2\ m$, $y = 0 \rightarrow 0 = -40(2)^2 + C_1(2) + C_2 \rightarrow 2C_1 + C_2 = 160$

At $x = 7\ m$, $y = 0 \rightarrow 0 = -40(7)^2 + 20\,(0)^3/3 + C_1(7) + C_2 \rightarrow 7C_1 + C_2 = 1960$

$\rightarrow$ $C_1 = 360$ and $C_2 = -560$

So, the general equation of the deflection y at any distance x is,

$$EIy = -40\,x^2 + 20\,(x-7)^3/3 + 360\,x - 560$$

(a) The deflection at A ($x = 0$)

$EIy_A = -40\,(0)^2 + 20\,(0-7)^3/3 + 360\,(0) - 560 = -560$

$y_A = -560/50000 = -0.0112\ m = -11.2\ mm$ **$y_A = 11.2\ mm \downarrow$**

The deflection at C ($x = 4.5\ m$)

$EIy_C = -40\,(4.5)^2 + 20\,(4.5-7)^3/3 + 360\,(4.5) - 560 = 250$

$y_C = 250/50000 = 0.005\ m = 5\ mm$ **$y_C = 5\ mm \uparrow$**

The deflection at E ($x = 9\ m$)

$EIy_E = -40\,(9)^2 + 20\,(9-7)^3/3 + 360\,(9) - 560 = -1520/3 = -506.6667$

$y_E = -1520/3/50000 = -0.010133\ m = -10.13\ mm$ **$y_E = 10.1\ mm \downarrow$**

(b) The slop at A ($x = 0$)

$EIy'_A = -80\,x + 20\,(x-7)^2 + C_1 = 0 + 0 + 360 = 360$

$y'_A = 360/50000 =$ **0.0072 rad** $= 0.41^\circ$

The slop at C ($x = 4.5\ m$)

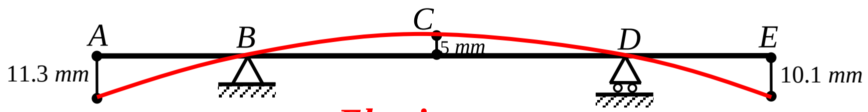
$EIy'_C = -80\,x + 20\,(x-7)^2 + C_1 = -360 + 0 + 360 = 0$

$y'_A = 0/50000 =$ **0**

The slop at E ($x = 9\ m$)

$EIy'_C = -80\,x + 20\,(x-7)^2 + C_1 = -720 + 80 + 360 = 0$

$y'_E = -280/50000 =$ **-0.0056 rad**



Elastic curve

With my best wishes
Dr. M. Abdel-Kader

Question (2): (14 Marks)

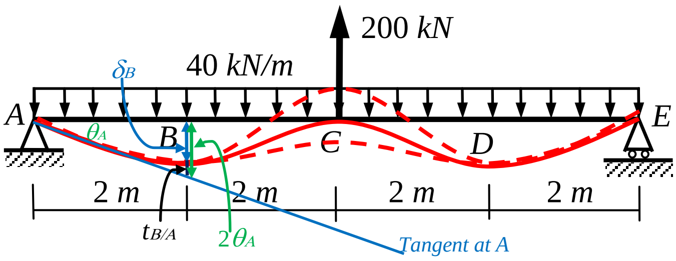
For the shown beam, using the **moment-area method**, determine:

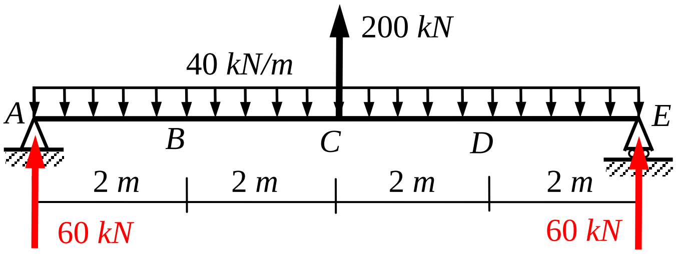
(a) the slope at ***A***

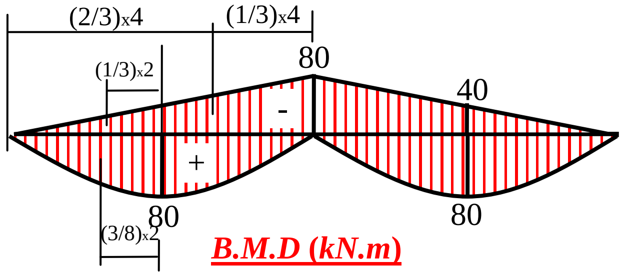
(b) the deflections at ***C*** and ***B***

and sketch the elastic curve of the beam.

$EI = 1\times10^4\ kN.m^2$







B.M.D (kN.m)

(a) From symmetry, the slope at ***C*** (θ_C) is equal to zero, the change in slope between the tangents of the elastic curve at points *A* and *C* (θ_{AC}) is equal to the slope at ***A*** (θ_A),

$$\theta_{AC} = \theta_C - \theta_A = 0 - \theta_A = -\theta_A$$

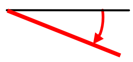
then

$$-\theta_A = \frac{1}{EI}\left[Area_{AC}\right]$$

$$= \frac{1}{10000}\left[\tfrac{2}{3}\times4\times80 - \tfrac{1}{2}\times4\times80\right] = \frac{160}{3\times10000}$$

$$= 0.00533\ rad$$

$\therefore$ $\boxed{\theta_A = -0.00533\ rad}$



(b) **The deflection at *C*** (δ_C) $= 4\theta_A - t_{C/A}$

$$t_{C/A} = \frac{1}{EI}\left[Area_{AC}.\bar{X}_C\right] = \frac{1}{10000}\left[(\tfrac{2}{3}\times4\times80)(2) - (\tfrac{1}{2}\times4\times80)(\tfrac{1}{3}\times4)\right] = 8/375 = 0.02133\ m$$

$$\delta_C = 4\theta_A - t_{C/A} = 4(0.00533) - 0.02133 =$$

$\therefore$ $\boxed{\delta_C = 0}$

OR

$$\delta_C = t_{A/C} = \frac{1}{EI}\left[Area_{AC}.\bar{X}_A\right] = \frac{1}{10000}\left[(\tfrac{2}{3}\times4\times80)(2) - (\tfrac{1}{2}\times4\times80)(\tfrac{2}{3}\times4)\right] = 0$$

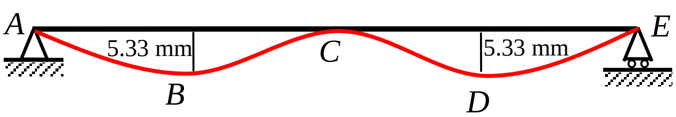
$\therefore$ $\boxed{\delta_C = 0}$

The deflection at *B* (δ_B) $= 2\theta_A - t_{B/A}$

$$t_{B/A} = \frac{1}{EI}\left[Area_{AB}.\bar{X}_B\right] = \frac{1}{10000}\left[(\tfrac{2}{3}\times2\times80)(\tfrac{3}{8}\times2) - (\tfrac{1}{2}\times2\times40)(\tfrac{1}{3}\times2)\right] = 2/375 = 0.00533$$

$$\delta_B = 2\theta_A - t_{B/A} = 2(0.00533) - 0.00533 = 0.00533\ m = 5.33\ mm \downarrow$$

$\therefore$ $\boxed{\delta_B = 5.33\ mm \downarrow}$



Elastic Curve

With my best wishes

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Question (3): (14 Marks)

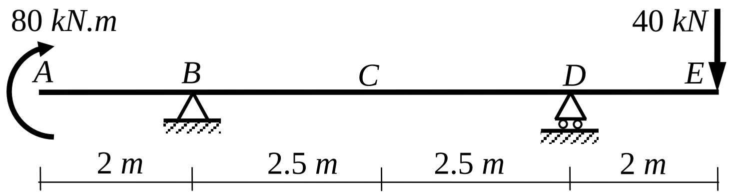
For the shown beam, using the **conjugate beam method**, determine:

(a) the deflections at ***A***, ***C*** and ***E***

(b) the slopes at ***A*** and ***C***

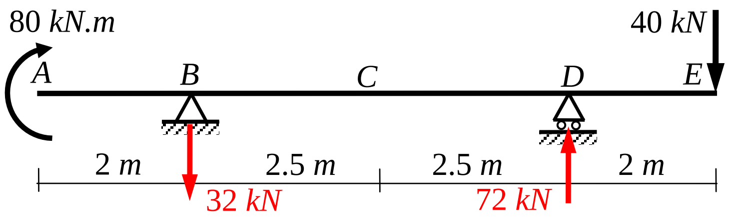
and sketch the elastic curve of the beam.

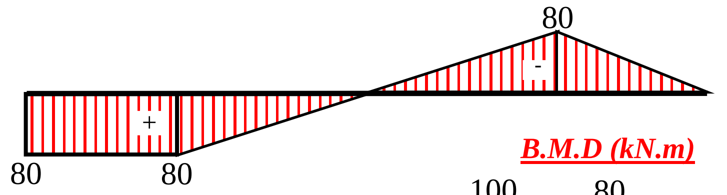
$EI = 5 \times 10^4\ kN.m^2$



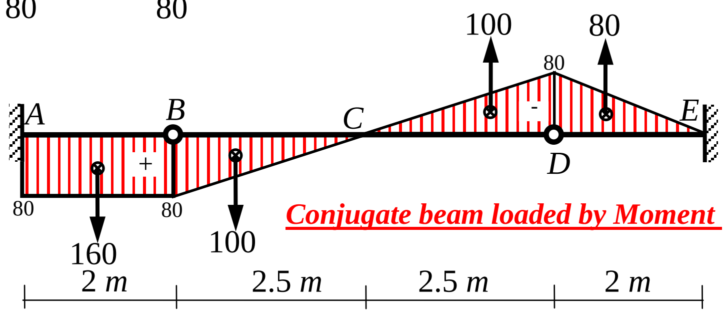
The bending moment diagram is as shown. The resulting moment diagram is then loaded to the conjugate beam.

For the conjugate beam, determine the elastic reaction (r_A and r_B).





To determine the elastic reaction at B (R_B) take the moment about D for part BD:



→ $R_B = 200/3$ ↑

Then $R_D = 200/3$ ↓

Deflection at *A* $= M_A / EI = -293.33 / 50000$

$\delta_A = -293.33 / 50000 = -0.005867\ m$

∴ $\delta_A = 5.87\ mm$ ↑

Deflection at *C* $= M_C / EI$

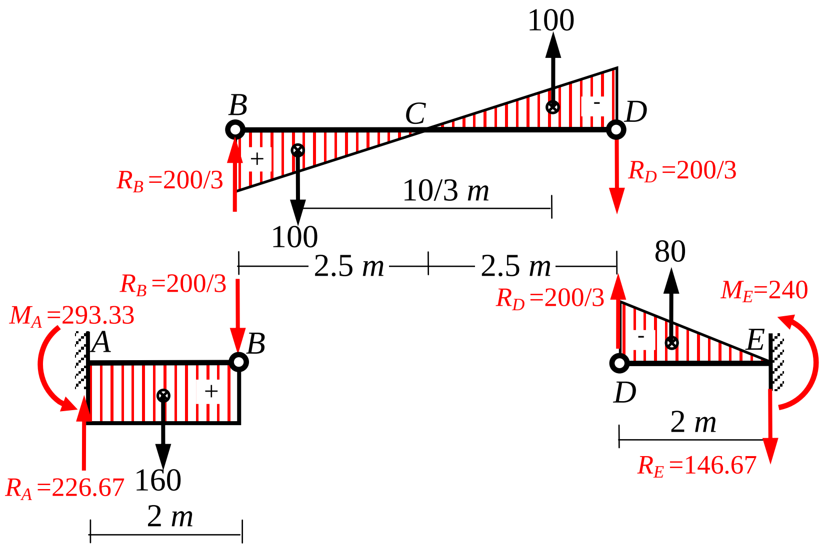
$= [200/3(2.5)-100(5/3)] / 50000$

$\delta_C = 0$ ∴ $\delta_C = 0$

Deflection at *E* $= M_E / EI = 240 / 50000$

$\delta_E = 240 / 50000 = -0.0048\ m$

∴ $\delta_E = 4.8\ mm$ ↑

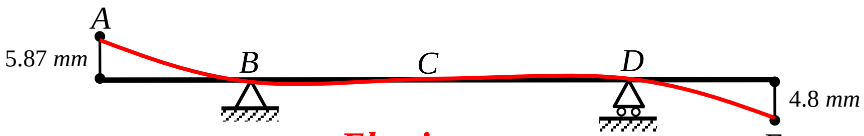


Slope at A = *Shear at A / EI* = 226.67 / 50000

= 0.00453 $\theta_A = 0.00453$ rad

Slope at C = *Shear at C / EI* = [200/3-100] / 50000

= [-33.33] / 50000 = -0.00067 $\theta_C = -0.00067$ rad



Elastic curve

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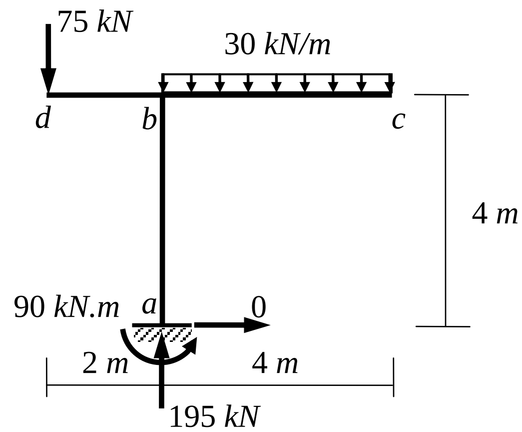
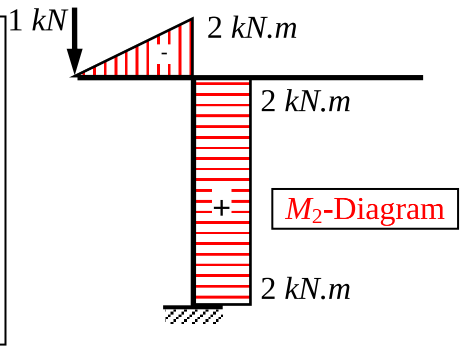
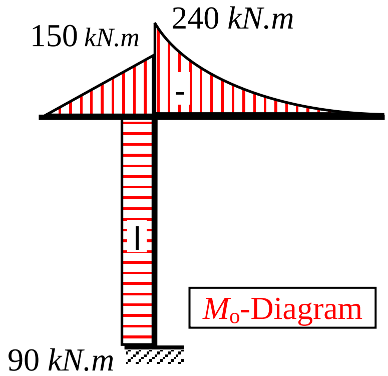
Question (4): (12 Marks)

For the shown frame and truss, using the **virtual work method**, determine the vertical displacement at d (δ_{dv}).

For the frame, the relative moments of inertia are given between brackets and $EI=20\times10^3\ kN.m^2$. For the truss, assume that all members have the same axial rigidity $EA=1000\ kN$.

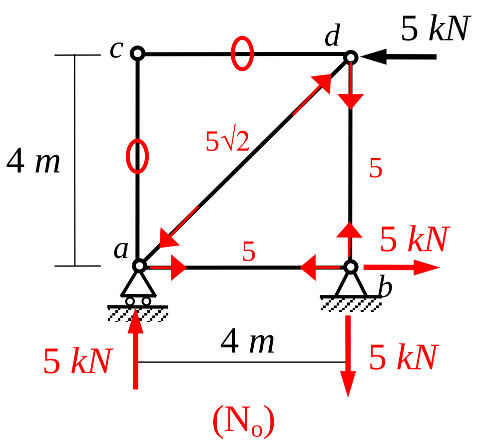


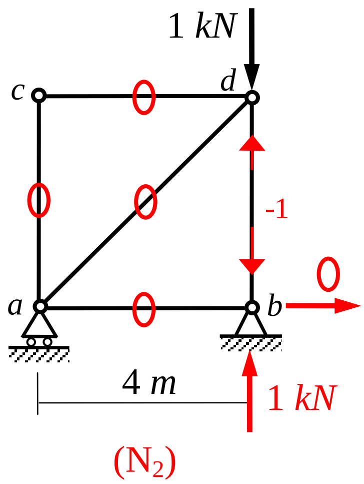




$$\delta_{dv} = \int \frac{M_o M_2}{EI} dL = \frac{1}{EI}\left[(-4\times90)(2)\right] + \frac{1}{EI}\left[(-\tfrac{1}{2}\times2\times150)(-\tfrac{2}{3}\times2)\right] = \frac{-520}{EI}\ \therefore$$

$$\delta_{dv} = -\frac{520}{20000} = 0.026\ m = 26\ mm \uparrow$$





Reactions and member forces due to:

(a) Applied load (b) Horizontal unit load at d (c) Vertical unit load at d

Calculation details for horizontal and vertical deflection of joint d

Member	EA (kN)	L (m)	N_o (kN)		N_2 (kN)		N_oN_2L / EA
1(ac)	1000	4	0		0		0
2(cd)	1000	4	0		0		0
3(bd)	1000	4	5		-1		-0.02
4(ab)	1000	4	5		0		0
5(ad)	1000	4√2	-5√2		0		0
Σ							δ_{dv}= -0.02 m (= 20 mm Upwards)

Vertical deflection of joint d $= \delta_d^v = \sum \frac{N_o N_2 L}{EA} = 20\ mm \uparrow$

With my best wishes

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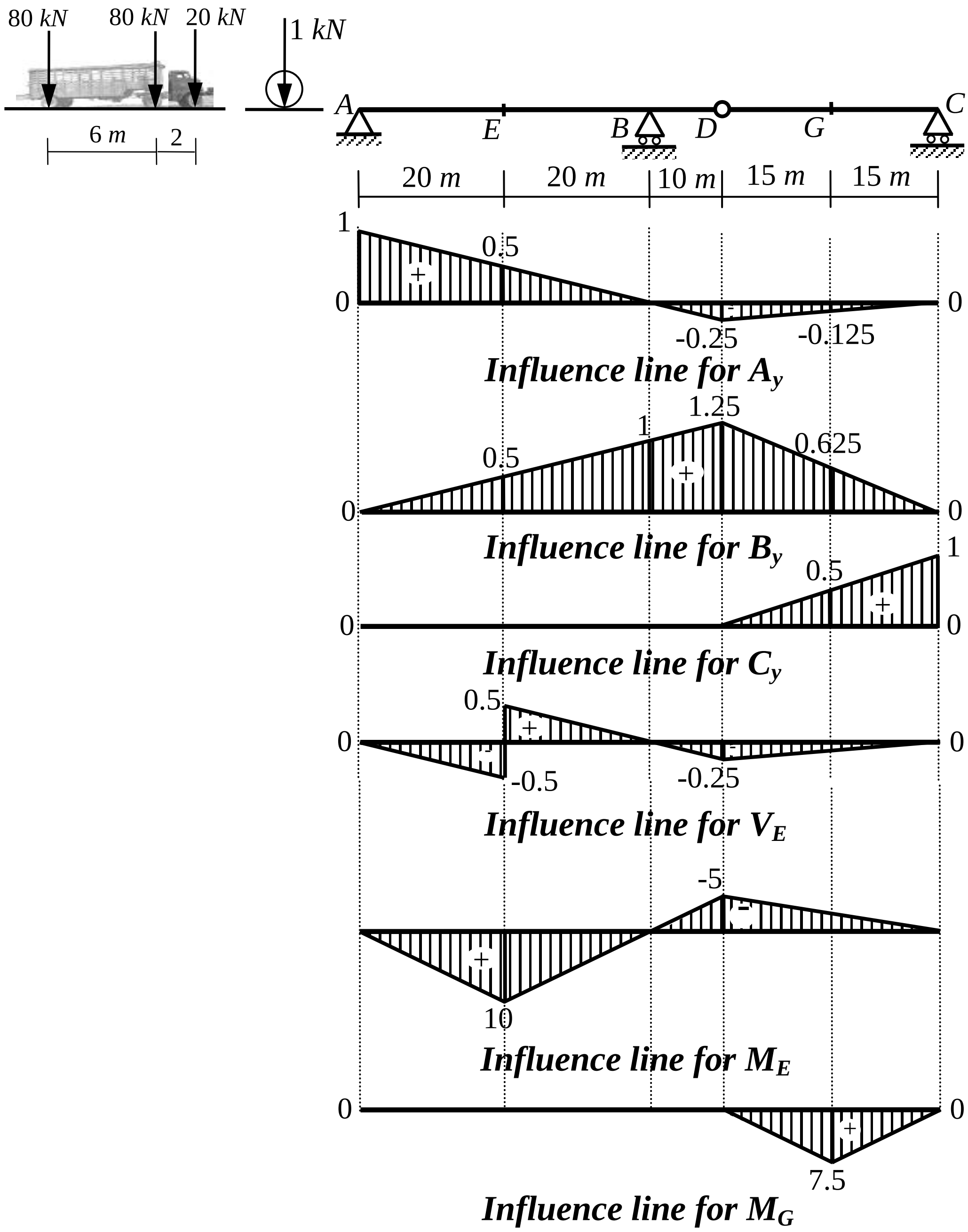
Question (5): (12 Marks)

For the shown beam, draw the influence line for:

(a) the reactions A_y, B_y and C_y.

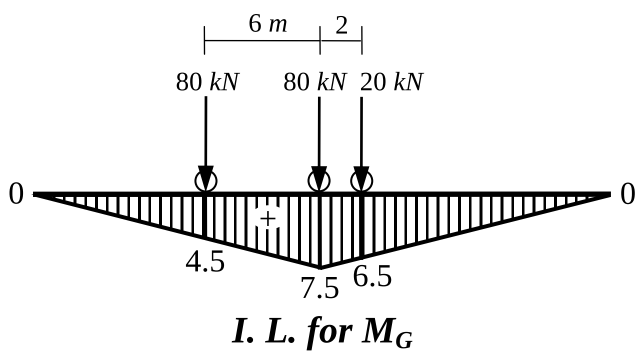
(b) the shear force at the section E and the bending moments at the sections E and G.

Also, determine the maximum moment at G caused by the shown moving truck.



$$M_{G\,max} = 80(4.5) + 80(7.5) + 20(6.5)$$
$$= 1090\ kN.m$$

$\therefore$ $M_{G\,max} = 1090\ kN.m$



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