

## Answer of Final Exam

Total Marks: **60**

No. of Questions: **3** (Attempt all questions)

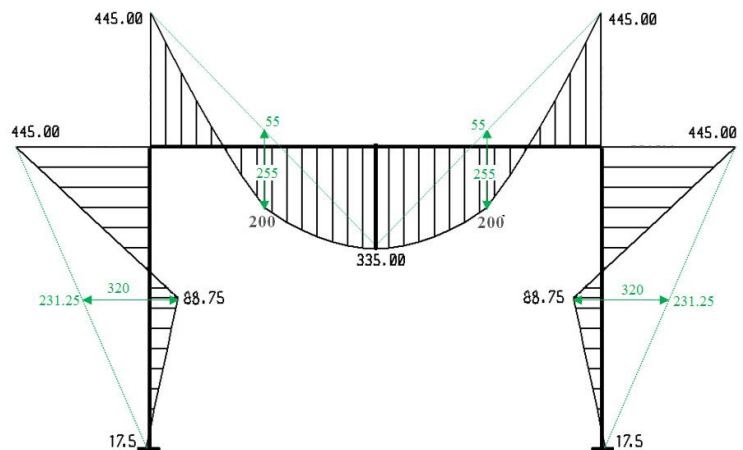
**Question (1): (10 Marks) (a2, a5)**

(a) Choose the correct answer (Put a, b, c or d in front of the statement number in your answer paper).

1.	c
2.	a
3.	d
4.	d
5.	b

(b) TRUE or FALSE (Put ✓ or ✗ in front of the statement number in your answer paper)

1.	✗
2.	✓
3.	✓
4.	✓
5.	✓
6.	✗
7.	✗
8.	✗
9.	✗
10.	✗
11.	✗
12.	✗
13.	✓
14.	✓
15.	✓



**Bending Moment Diagram**

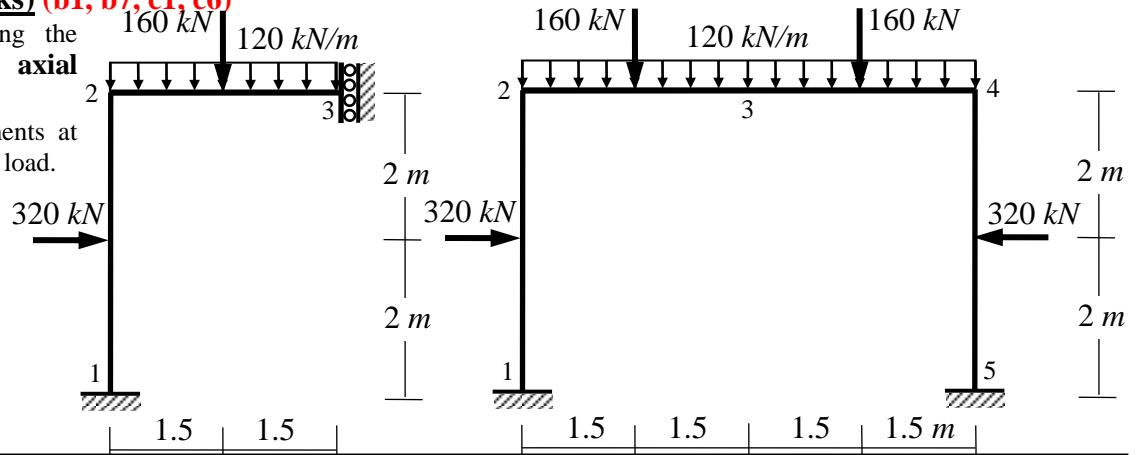
**Question (2): (20 Marks) (b1, b7, c1, c6)**

For the shown frame, using the stiffness method: **Neglect axial deformation**

- (a) Determine the displacements at the nodes due to the given load.
- (b) Draw the bending moment diagram.

**Given Data:**

$E = 2.1 \times 10^7 \text{ kN/m}^2$   
 $A = 0.15 \text{ m}^2$   
 $I = 3.125 \times 10^{-3} \text{ m}^4$



**Element (1): (nodes 1 & 2)**

$\lambda = \cos \alpha = 0$  and  $\mu = \sin \alpha = 1$   
 $6EI/L^2 = 6 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 4^2 = 24609$   
 $4EI/L = 4 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 4 = 65625$   
 $2EI/L = 32813$

$$\begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ F_{x2} \\ F_{y2} \\ M_2 \end{Bmatrix} = \begin{bmatrix} - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \theta_2 \end{Bmatrix} + \begin{Bmatrix} -160 \\ 0 \\ 160 \\ -160 \\ 0 \\ -160 \end{Bmatrix}$$

**Element (2): (nodes 2 & 3)**

$\lambda = \cos \alpha = 1$  and  $\mu = \sin \alpha = 0$   
 $12EI/L^3 = 12 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 3^3 = 29167$   
 $6EI/L^2 = 6 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 3^2 = 43750$   
 $4EI/L = 4 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 3 = 87500$   
 $2EI/L = 43750$

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ M_2 \\ F_{x3} \\ F_{y3} \\ M_3 \end{Bmatrix} = \begin{bmatrix} - & - & 0 & - & 0 & - \\ - & - & 43750 & - & -29167 & - \\ - & - & 87500 & - & -43750 & - \\ - & - & 0 & - & 0 & - \\ - & - & -43750 & - & 29167 & - \\ - & - & 43750 & - & -43750 & - \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \\ v_3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 260 \\ 150 \\ 0 \\ 260 \\ -150 \end{Bmatrix}$$

**Frame equation**

$$\begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ 0 \\ 0 \\ 0 \\ X_3 \\ 0 \\ M_3 \end{Bmatrix} = \begin{bmatrix} - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \\ v_3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -160 \\ 0 \\ 160 \\ -160 \\ 260 \\ -10 \\ 0 \\ 260 \\ -150 \end{Bmatrix}$$

From Row No. 6  $\rightarrow 0 = (153125) (\theta_2) - 43750 (v_3) - 10$

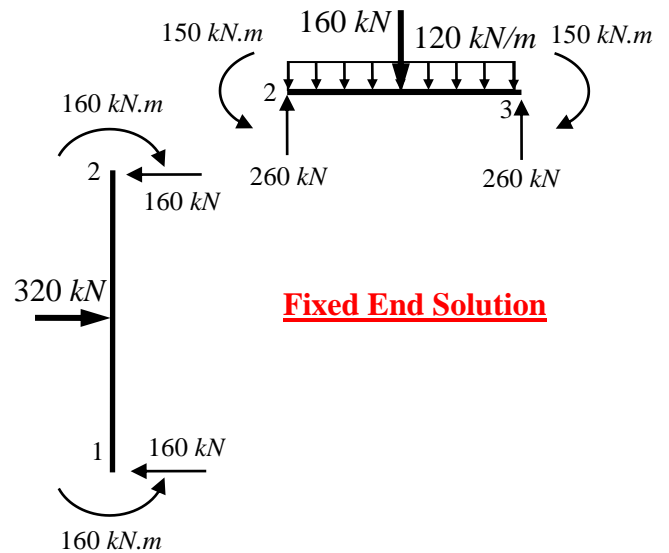
From Row No. 8  $\rightarrow 0 = -43750 (\theta_2) + (29167) (v_3) + 260 \rightarrow \theta_2 = -4.3428 \times 10^{-4} \text{ rad}$  and  $v_3 = -0.01543 \text{ m}$

**From Element 1**

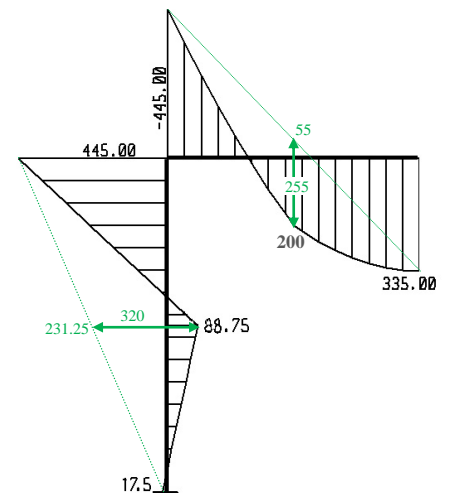
$M_1 = 32813 (-4.3428 \times 10^{-3}) + 160 = +17.5 \text{ kN.m}$   
 $M_2 = 65625 (-4.3428 \times 10^{-3}) - 160 = -445 \text{ kN.m}$

**From Element 2**

$M_2 = 87500 (-4.3428 \times 10^{-3}) - 43750 (-0.01543) + 150 = +445 \text{ kN.m}$   
 $M_3 = 43750 (-4.3428 \times 10^{-3}) - 43750 (-0.01543) - 150 = +335 \text{ kN.m}$



**Fixed End Solution**



**Bending Moment Diagram**

With my best wishes

Dr. M. Abdel-Kader

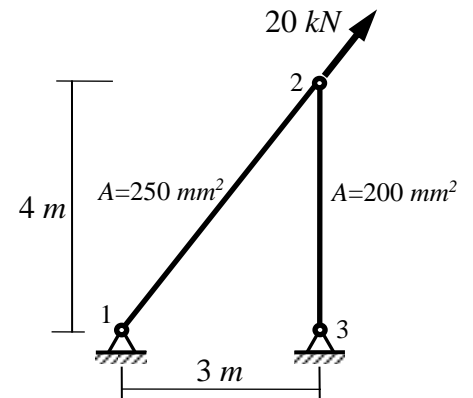
**Question (3): (20 Marks) (b1, b7, c1, c6)**

For the shown truss, using the stiffness method:

- (a) Determine the displacements at the nodes due to the given load.
- (b) Determine the reactions at the supports.

**Given Data:**  $E = 2.0 \times 10^7 \text{ kN/m}^2$ .

A for each member is as shown on the truss.



Truss (Bar) element

$$[K_e] = \begin{bmatrix} \frac{EA}{L} \lambda^2 & \frac{EA}{L} \mu \lambda & -\frac{EA}{L} \lambda^2 & -\frac{EA}{L} \mu \lambda \\ \frac{EA}{L} \mu \lambda & \frac{EA}{L} \mu^2 & -\frac{EA}{L} \mu \lambda & -\frac{EA}{L} \mu^2 \\ -\frac{EA}{L} \lambda^2 & -\frac{EA}{L} \mu \lambda & \frac{EA}{L} \lambda^2 & \frac{EA}{L} \mu \lambda \\ -\frac{EA}{L} \mu \lambda & -\frac{EA}{L} \mu^2 & \frac{EA}{L} \mu \lambda & \frac{EA}{L} \mu^2 \end{bmatrix}$$

Where,  $\lambda = \cos \alpha$  and  $\mu = \sin \alpha$

**Element (1): (nodes 1 & 2)**

$\lambda = \cos \alpha = 0.6$  and  $\mu = \sin \alpha = 0.8$

$EA/L = 2.0 \times 10^7 \times 2.5 \times 10^{-4} / 5 = 1000$

$$\begin{Bmatrix} X_1 \\ Y_1 \\ F_{x2} \\ F_{y2} \end{Bmatrix} = \begin{bmatrix} 360 & 480 & -360 & -480 \\ 480 & 640 & -480 & -640 \\ -360 & -384 & 360 & 480 \\ -480 & -640 & 480 & 640 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \end{Bmatrix}$$

**Element (2): (nodes 2 & 3)**

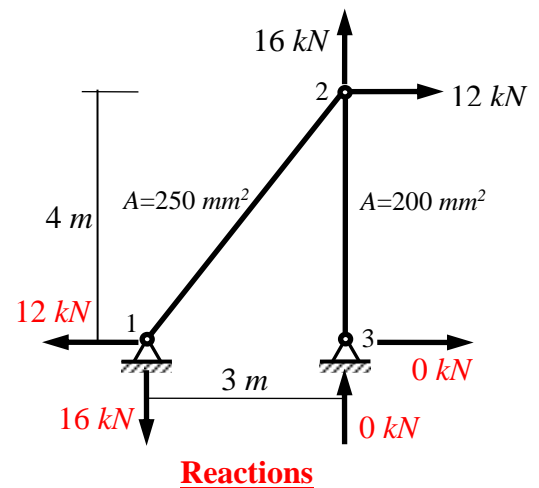
$\lambda = \cos \alpha = 0$  and  $\mu = \sin \alpha = -1$

$EA/L = 2.0 \times 10^7 \times 2.0 \times 10^{-4} / 4 = 1000$

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ X_3 \\ Y_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & -1000 \\ 0 & 0 & 0 & 0 \\ 0 & -1000 & 0 & 1000 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ 0 \\ 0 \end{Bmatrix}$$

**Truss equation**

	1	2	3		
$X_1$	360	480	-360	-480	0
$Y_1$	480	640	-480	-640	0
12	-360	-480	(360+0)	(480+0)	0
16	-480	-640	(480+0)	(640+1000)	-1000
$X_3$	0	0	0	0	0
$Y_3$	0	0	0	-1000	1000



**Reactions**

From Row No. 3  $\rightarrow 12 = (360)(u_2) + (480)(v_2)$

From Row No. 4  $\rightarrow 16 = (480)(u_2) + (1640)(v_2) \rightarrow \boxed{u_2 = 0.0333 = m}$  and  $\boxed{v_2 = 0 = m}$

From Row No. 1  $\rightarrow X_1 = -360(0.0333) - 384(0) = -12 \text{ kN}$

$\boxed{X_1 = 12 \text{ kN} \leftarrow}$

From Row No. 2  $\rightarrow Y_1 = -480(0.0333) - 512(0) = -16 \text{ kN}$

$\boxed{Y_1 = 16 \text{ kN} \downarrow}$

From Row No. 5  $\rightarrow X_3 = 0(-0.0333) + 0(0) = 0$

$\boxed{X_3 = 0}$

From Row No. 6  $\rightarrow Y_3 = 0(-0.0333) - 1000(0) = 0$

$\boxed{Y_3 = 0}$

With my best wishes

Dr. M. Abdel-Kader