

## Answer of Final Exam

Total Marks: **60**

No. of Questions: **3** (Attempt all questions)

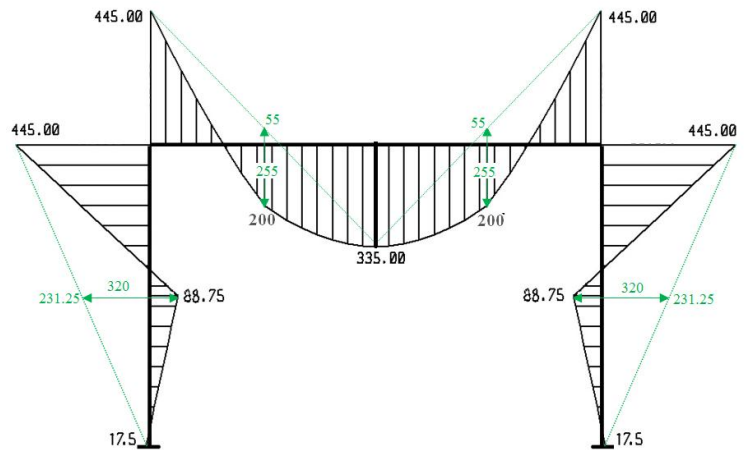
### Question (1): (10 Marks)

(a) Choose the correct answer (Put a, b, c or d in front of the statement number in your answer paper).

|    |   |
|----|---|
| 1. | c |
| 2. | a |
| 3. | d |
| 4. | d |
| 5. | a |

(b) TRUE or FALSE (Put ✓ or ✗ in front of the statement number in your answer paper)

|     |   |
|-----|---|
| 1.  | ✗ |
| 2.  | ✓ |
| 3.  | ✓ |
| 4.  | ✓ |
| 5.  | ✓ |
| 6.  | ✗ |
| 7.  | ✗ |
| 8.  | ✗ |
| 9.  | ✗ |
| 10. | ✗ |
| 11. | ✗ |
| 12. | ✗ |
| 13. | ✓ |
| 14. | ✓ |
| 15. | ✓ |



**Bending Moment Diagram**

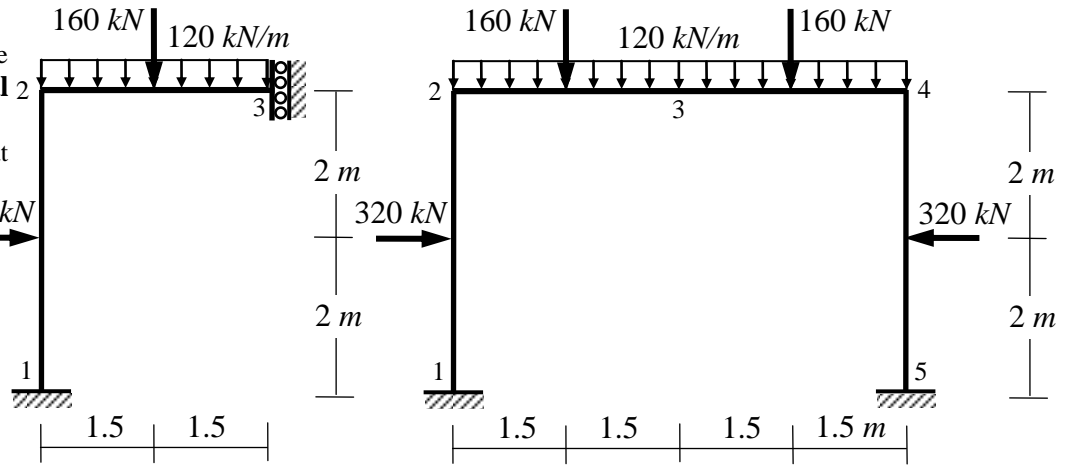
**Question (2): (20 Marks)**

For the shown frame, using the stiffness method: **Neglect axial deformation**

- (a) Determine the displacements at the nodes due to the given load.
- (b) Draw the bending moment diagram.

**Given Data:**

$E = 2.1 \times 10^7 \text{ kN/m}^2$   
 $A = 0.15 \text{ m}^2$   
 $I = 3.125 \times 10^{-3} \text{ m}^4$



**Element (1): (nodes 1 & 2)**

$\lambda = \cos \alpha = 0$  and  $\mu = \sin \alpha = 1$   
 $6EI/L^2 = 6 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 4^2 = 24609$   
 $4EI/L = 4 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 4 = 65625$   
 $2EI/L = 32813$

$$\begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ F_{x2} \\ F_{y2} \\ M_2 \end{Bmatrix} = \begin{bmatrix} - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \\ - & - & - & - & - & - \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -160 \\ 0 \\ 160 \\ -160 \\ 0 \\ -160 \end{Bmatrix}$$

**Element (2): (nodes 2 & 3)**

$\lambda = \cos \alpha = 1$  and  $\mu = \sin \alpha = 0$   
 $12EI/L^3 = 12 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 3^3 = 29167$   
 $6EI/L^2 = 6 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 3^2 = 43750$   
 $4EI/L = 4 \times 2.1 \times 10^7 \times 3.125 \times 10^{-3} / 3 = 87500$   
 $2EI/L = 43750$

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ M_2 \\ F_{x3} \\ F_{y3} \\ M_3 \end{Bmatrix} = \begin{bmatrix} - & - & 0 & - & 0 & - \\ - & - & 43750 & - & -29167 & - \\ - & - & 87500 & - & -43750 & - \\ - & - & 0 & - & 0 & - \\ - & - & -43750 & - & 29167 & - \\ - & - & 43750 & - & -43750 & - \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \\ v_3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 260 \\ 150 \\ 0 \\ 260 \\ -150 \end{Bmatrix}$$

**Frame equation**

$$\begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ 0 \\ 0 \\ 0 \\ X_3 \\ 0 \\ M_3 \end{Bmatrix} = \begin{bmatrix} - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \\ - & - & - & - & - & - & - & - & - \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \\ v_3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} -160 \\ 0 \\ 160 \\ -160 \\ 260 \\ -10 \\ 0 \\ 260 \\ -150 \end{Bmatrix}$$

From Row No. 6  $\rightarrow 0 = (153125) (\theta_2) - 43750 (v_3) - 10$

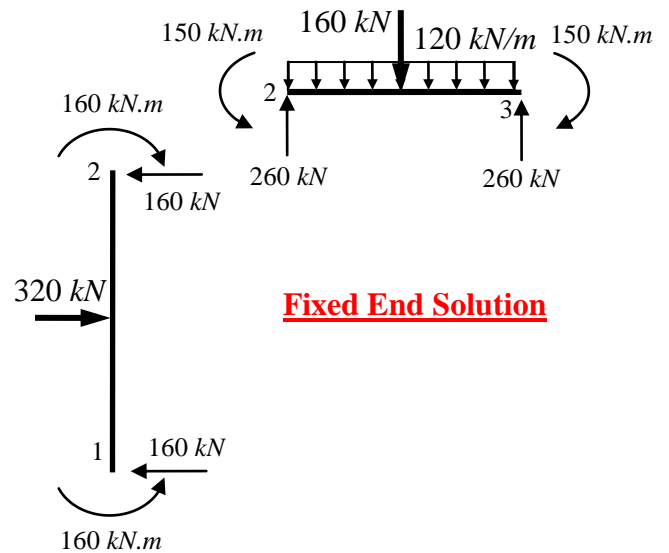
From Row No. 8  $\rightarrow 0 = -43750 (\theta_2) + (29167) (v_3) + 260 \rightarrow \theta_2 = -4.3428 \times 10^{-4} \text{ rad}$  and  $v_3 = -0.01543 \text{ m}$

**From Element 1**

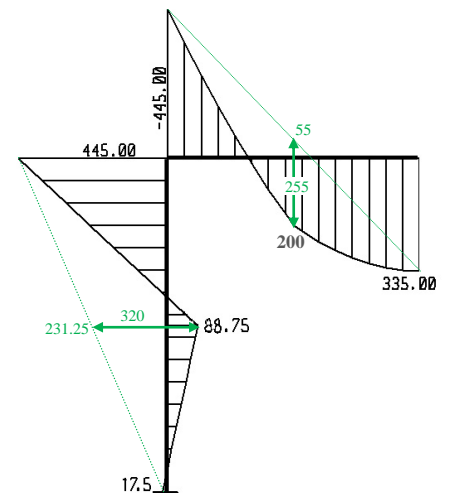
$M_1 = 32813 (-4.3428 \times 10^{-3}) + 160 = +17.5 \text{ kN.m}$   
 $M_2 = 65625 (-4.3428 \times 10^{-3}) - 160 = -445 \text{ kN.m}$

**From Element 2**

$M_2 = 87500 (-4.3428 \times 10^{-3}) - 43750 (-0.01543) + 150 = +445 \text{ kN.m}$   
 $M_3 = 43750 (-4.3428 \times 10^{-3}) - 43750 (-0.01543) - 150 = +335 \text{ kN.m}$



**Fixed End Solution**



**Bending Moment Diagram**

With my best wishes

Dr. M. Abdel-Kader

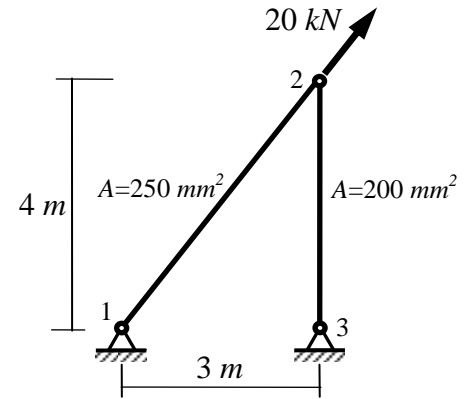
**Question (3): (20 Marks)**

For the shown truss, using the stiffness method:

- (a) Determine the displacements at the nodes due to the given load.
- (b) Determine the reactions at the supports.

**Given Data:**  $E = 2.0 \times 10^7 \text{ kN/m}^2$ .

A for each member is as shown on the truss.



Truss (Bar) element

$$[K_e] = \begin{bmatrix} \frac{EA}{L} \lambda^2 & \frac{EA}{L} \mu \lambda & -\frac{EA}{L} \lambda^2 & -\frac{EA}{L} \mu \lambda \\ \frac{EA}{L} \mu \lambda & \frac{EA}{L} \mu^2 & -\frac{EA}{L} \mu \lambda & -\frac{EA}{L} \mu^2 \\ -\frac{EA}{L} \lambda^2 & -\frac{EA}{L} \mu \lambda & \frac{EA}{L} \lambda^2 & \frac{EA}{L} \mu \lambda \\ -\frac{EA}{L} \mu \lambda & -\frac{EA}{L} \mu^2 & \frac{EA}{L} \mu \lambda & \frac{EA}{L} \mu^2 \end{bmatrix}$$

Where,  $\lambda = \cos \alpha$  and  $\mu = \sin \alpha$

**Element (1): (nodes 1 & 2)**

$\lambda = \cos \alpha = 0.6$  and  $\mu = \sin \alpha = 0.8$

$EA/L = 2.0 \times 10^7 \times 2.5 \times 10^{-4} / 5 = 1000$

$$\begin{Bmatrix} X_1 \\ Y_1 \\ F_{x2} \\ F_{y2} \end{Bmatrix} = \begin{bmatrix} 360 & 480 & -360 & -480 \\ 480 & 640 & -480 & -640 \\ -360 & -384 & 360 & 480 \\ -480 & -640 & 480 & 640 \end{bmatrix} \begin{Bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \end{Bmatrix}$$

**Element (2): (nodes 2 & 3)**

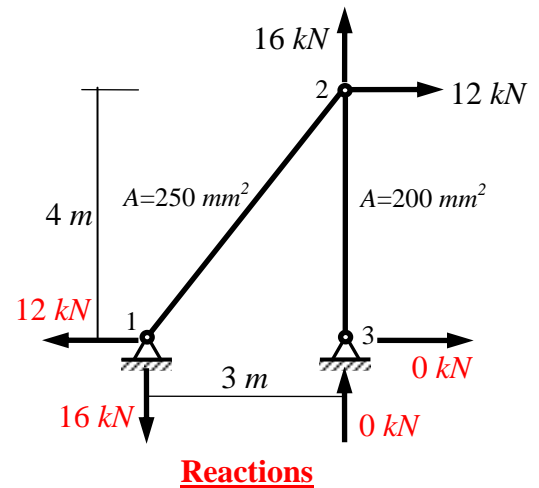
$\lambda = \cos \alpha = 0$  and  $\mu = \sin \alpha = -1$

$EA/L = 2.0 \times 10^7 \times 2.0 \times 10^{-4} / 4 = 1000$

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ X_3 \\ Y_3 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1000 & 0 & -1000 \\ 0 & 0 & 0 & 0 \\ 0 & -1000 & 0 & 1000 \end{bmatrix} \begin{Bmatrix} u_2 \\ v_2 \\ 0 \\ 0 \end{Bmatrix}$$

**Truss equation**

|       |      |      |         |            |   |       |  |
|-------|------|------|---------|------------|---|-------|--|
|       | 1    | 2    | 3       |            |   |       |  |
| $X_1$ | 360  | 480  | -360    | -480       | 0 | 0     | $\begin{Bmatrix} 0 \\ 0 \\ u_2 \\ v_2 \\ 0 \\ 0 \end{Bmatrix}$ |
| $Y_1$ | 384  | 640  | -384    | -640       | 0 | 0     |  |
| 12    | -360 | -480 | (360+0) | (480+0)    | 0 | 0     |  |
| 16    | -480 | -640 | (480+0) | (640+1000) | 0 | -1000 |  |
| $X_3$ | 0    | 0    | 0       | 0          | 0 | 0     |  |
| $Y_3$ | 0    | 0    | 0       | -1000      | 0 | 1000  |  |



From Row No. 3  $\rightarrow 12 = (360)(u_2) + (480)(v_2)$

From Row No. 4  $\rightarrow 16 = (480)(u_2) + (1640)(v_2) \rightarrow u_2 = 0.0333 = m$  and  $v_2 = 0 = m$

From Row No. 1  $\rightarrow X_1 = -360(0.0333) - 384(0) = -12 \text{ kN}$

$X_1 = 12 \text{ kN} \leftarrow$

From Row No. 2  $\rightarrow Y_1 = -480(0.0333) - 512(0) = -16 \text{ kN}$

$Y_1 = 16 \text{ kN} \downarrow$

From Row No. 5  $\rightarrow X_3 = 0(-0.0333) + 0(0) = 0$

$X_3 = 0$

From Row No. 6  $\rightarrow Y_3 = 0(-0.0333) - 1000(0) = 0$

$Y_3 = 0$

With my best wishes

Dr. M. Abdel-Kader