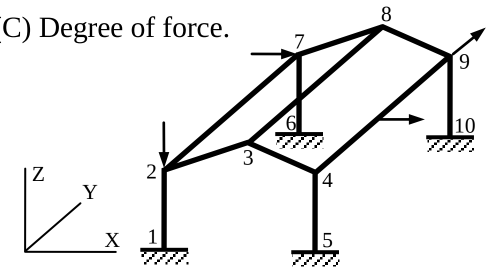
# Final Exam

Total Marks: **60** No. of Questions: **3** (Attempt all questions)

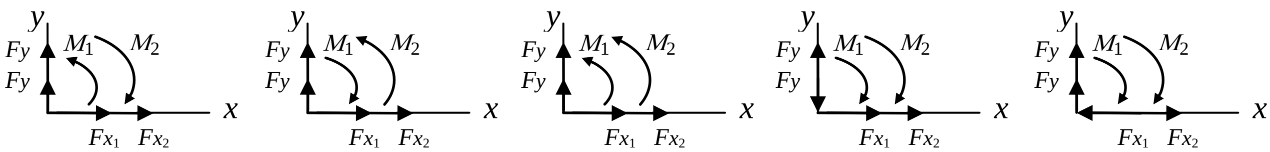
## Question (1): (20 Marks)

Choose the nearest correct answer (**Put A, B, C, D or E in front of the statement number in your answer paper**).

1. The responsibility of the analytical model results lies on:
   (A) The company developed the software. (B) The computer used. (C) The input data.
   (D) The structural designer who used the software. (E) The structural designer who developed the software.
2. The abbreviation "CAD" means:
   (A) Common Analysis Data. (B) Computer And Data. (C) Calculation And Design.
   (D) Computer-Aided Design. (E) Computer-Aided Data.
3. The abbreviation "DOF" means:
   (A) Possible translations at nodes. (B) Possible rotations at nodes. (C) Degree of force.
   (D) Possible displacements at nodes. (E) Deformation of forces.

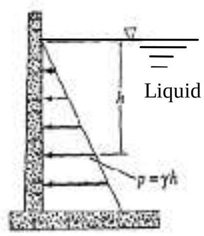



4. The number of non-zero DOF for the shown space frame is:
   (A) 60 (B) 30 (C) 36 (D) 18 (E) 10
5. The number of non-zero DOF per node 1 in the shown space frame is:
   (A) Zero (B) 1 (C) 2 (D) 3 (E) 6
6. The number of non-zero DOF per node 2 in the shown space frame is:
   (A) Zero (B) 3 (C) 4 (D) 6 (E) 5
7. If the axial deformation is neglected, the number of non-zero DOF per node 2 in the shown space frame is:
   (A) 4 (B) 3 (C) 5 (D) 2 (E) 6
8. When there are loads between the nodes, the equilibrium equation of a plane frame is $\{F\} = [K]\{\Delta\}+\{F^f\}$ where;
   (A) $\{F\}$ is the nodal forces. (B) $\{F^f\}$ is the nodal displacements. (C) $[F^f]$ is the element stiffness matrix.
   (D) $\{\Delta\}$ is the nodal forces. (E) $[K]$ is square of an order equal to the number of members.
9. In 2D Analysis, ………………. can be used.
   (A) only 2D elements (B) 1D, 2D and 3D elements (C) 2D and 3D elements
   (D) 1D and 2D elements (E) 1D and 3D elements
10. Structures that cannot be modeled with the frame element are:
    (A) Space frames. (B) Space trusses. (C) Flat slabs. (D) Plane frames. (E) Beams.
11. Stiffness is the property of an element which is defined as:
    (A) Displacement per unit area. (B) Displacement per unit force. (C) Force per unit displacement.
    (D) Force per unit displacement. (E) Force per unit mass.
12. For plane frame element 1-2 (connecting joints 1 and 2), the positive sign of forces (forces and moments) is:




(A) (B) (C) (D) (E)

13. One of the assumptions that the stiffness method is based on to analyze plane frames is:
    (A) Members will behave in non-linear and plastic manner.
    (B) Members (beams and columns) are straight with variable properties between joints.
    (C) Axial forces in members are very much less than the respective Euler buckling loads.
    (D) Applied loads may act out of the structure plane.
    (E) Deflections are sufficiently large for the changes of geometry to be ignored.
14. In SAP, loads and properties of material are:
    (A) Fixed data. (B) Output data. (C) Input data. (D) Not important.
    (E) Results of the analysis
15. In SAP, the ………. must be defined for (1D) plane frame elements.
    (A) Areas (B) Thicknesses (C) Sections (D) Volumes (E) Colors




16. The triangle load applied on the shown vertical wall is called ………. load.
    (A) Earth pressure (B) Earthquake (C) Settlement (D) Temperature (E) Hydrostatic

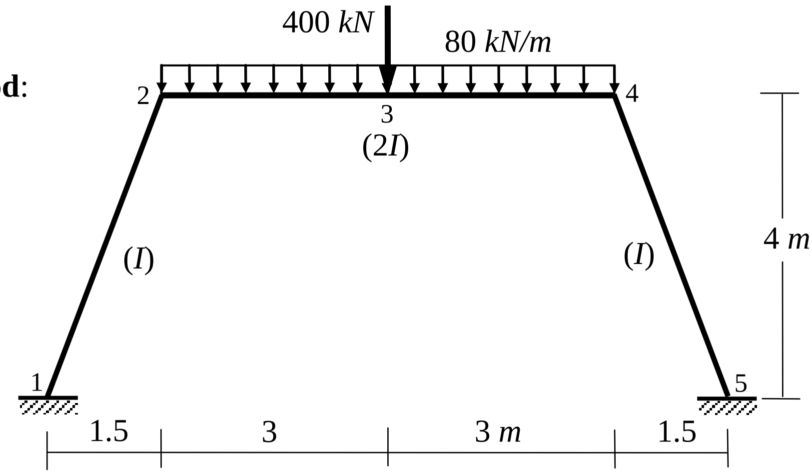
***Please turn over***

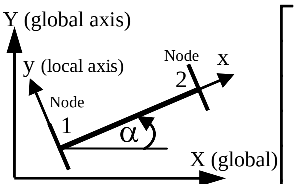
17. In SAP, loads and properties of material are:
    (A) Fixed data. (B) Output data. (C) Results of the analysis. (D) Not important. (E) Input data.
18. In SAP, self-weight loading always acts in the … direction.
    (A) X (B) Y (C) –Y (D) Z (E) –Z
19. Wind load applied over the height of high rise buildings is assumed:
    (A) Constant. (B) 1000 *kN/m*. (C) Zero. (D) Parallel to the surface. (E) Perpendicular to the surface.
20. When the material properties are independent of the coordinates, the material is:
    (A) Isotropic. (B) Non-linear. (C) Plastic. (D) Wood. (E) Homogeneous.

## Question (2): (20 Marks)

For the shown frame, **using the stiffness method**:
**Neglect axial deformation**
(a) Determine the displacements at the nodes due to the given loads.
(b) Draw the bending moment diagram.

Note that *E* and *A* are constants, and the relative moments of inertia are given between brackets.



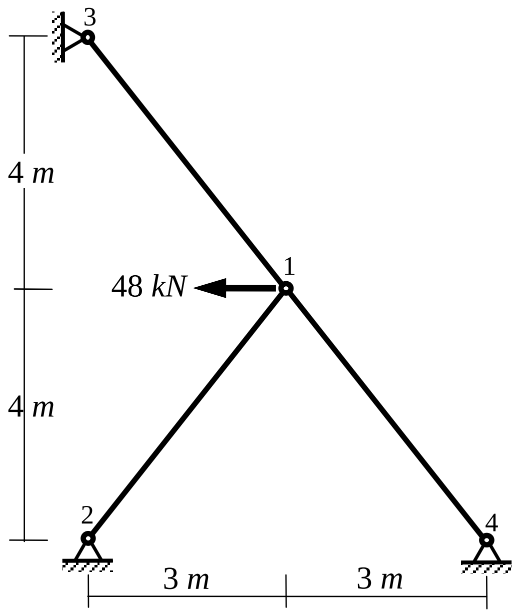

***Frame element***

$$[K_e]=\begin{bmatrix}
\left(\frac{EA}{L}\lambda^2+\frac{12EI}{L^3}\mu^2\right) & \left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & -\frac{6EI}{L^2}\mu & \left(-\frac{EA}{L}\lambda^2-\frac{12EI}{L^3}\mu^2\right) & \left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & -\frac{6EI}{L^2}\mu \\
\left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & \left(\frac{EA}{L}\mu^2+\frac{12EI}{L^3}\lambda^2\right) & \frac{6EI}{L^2}\lambda & \left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & \left(-\frac{EA}{L}\mu^2-\frac{12EI}{L^3}\lambda^2\right) & \frac{6EI}{L^2}\lambda \\
-\frac{6EI}{L^2}\mu & \frac{6EI}{L^2}\lambda & \frac{4EI}{L} & \frac{6EI}{L^2}\mu & -\frac{6EI}{L^2}\lambda & \frac{2EI}{L} \\
\left(-\frac{EA}{L}\lambda^2-\frac{12EI}{L^3}\mu^2\right) & \left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & \frac{6EI}{L^2}\mu & \left(\frac{EA}{L}\lambda^2+\frac{12EI}{L^3}\mu^2\right) & \left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & \frac{6EI}{L^2}\mu \\
\left(-\frac{EA}{L}\mu\lambda+\frac{12EI}{L^3}\mu\lambda\right) & \left(-\frac{EA}{L}\mu^2-\frac{12EI}{L^3}\lambda^2\right) & -\frac{6EI}{L^2}\lambda & \left(\frac{EA}{L}\mu\lambda-\frac{12EI}{L^3}\mu\lambda\right) & \left(\frac{EA}{L}\mu^2+\frac{12EI}{L^3}\lambda^2\right) & -\frac{6EI}{L^2}\lambda \\
-\frac{6EI}{L^2}\mu & \frac{6EI}{L^2}\lambda & \frac{2EI}{L} & \frac{6EI}{L^2}\mu & -\frac{6EI}{L^2}\lambda & \frac{4EI}{L}
\end{bmatrix}$$

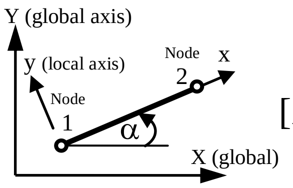
Where, $\lambda = \cos\alpha$ and $\mu = \sin\alpha$

## Question (3): (20 Marks)

For the shown truss, **using the stiffness method**:
(a) Determine the displacements at the nodes due to the given loads.
(b) Determine the reactions at the supports.
(c) Determine the forces in the members.




**Given Data:**
*EA* is constant for all members.




***Truss (Bar) element***

$$[K_e]=\begin{bmatrix}
\frac{EA}{L}\lambda^2 & \frac{EA}{L}\mu\lambda & -\frac{EA}{L}\lambda^2 & -\frac{EA}{L}\mu\lambda \\
\frac{EA}{L}\mu\lambda & \frac{EA}{L}\mu^2 & -\frac{EA}{L}\mu\lambda & -\frac{EA}{L}\mu^2 \\
-\frac{EA}{L}\lambda^2 & -\frac{EA}{L}\mu\lambda & \frac{EA}{L}\lambda^2 & \frac{EA}{L}\mu\lambda \\
-\frac{EA}{L}\mu\lambda & -\frac{EA}{L}\mu^2 & \frac{EA}{L}\mu\lambda & \frac{EA}{L}\mu^2
\end{bmatrix}$$

Where, $\lambda = \cos\alpha$ and $\mu = \sin\alpha$

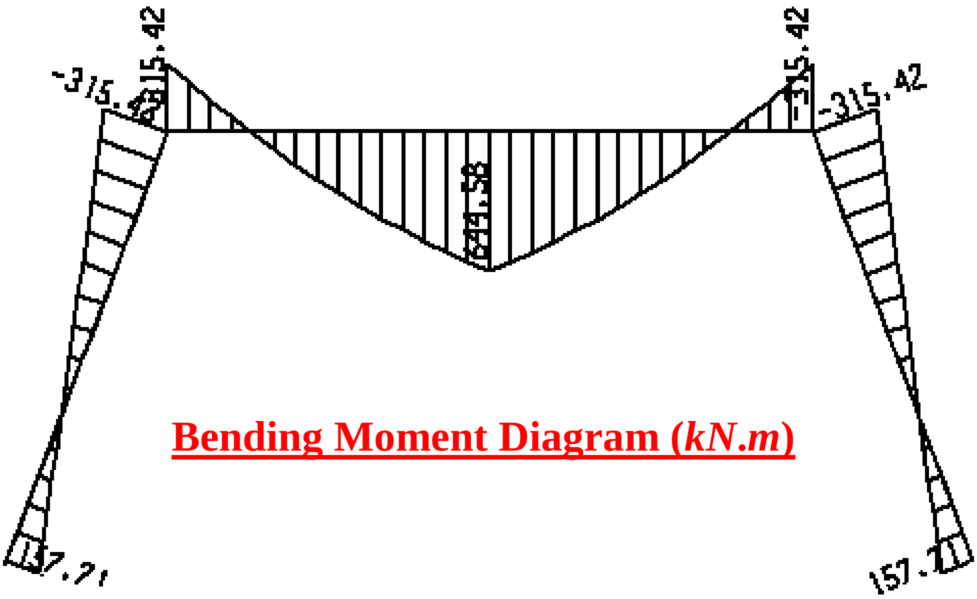
*With my best wishes*

***Dr. M. Abdel-Kader***

**Question (1): (10 Marks)**

Choose the correct answer (Put A, B, C, D or E in front of the statement number in your answer paper).

| | | | | |
|---|---|---|---|---|
| 1. | D | | 11. | C or D |
| 2. | D | | 12. | C |
| 3. | D | | 13. | C |
| 4. | C | | 14. | C |
| 5. | A | | 15. | C |
| 6. | D | | 16. | E |
| 7. | C | | 17. | E Repeated |
| 8. | A | | 18. | E |
| 9. | D | | 19. | E |
| 10. | C | | 20. | E |




**Bending Moment Diagram (*kN.m*)**

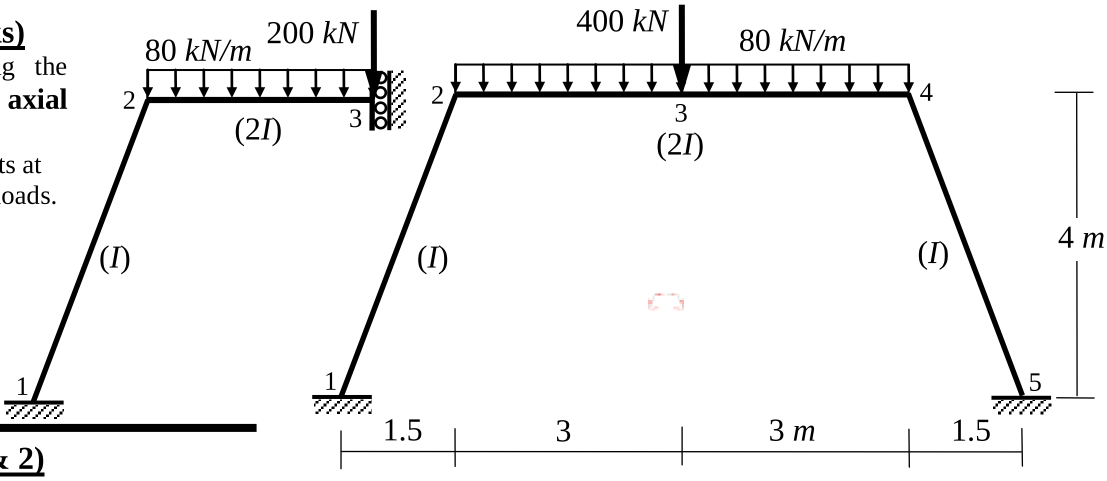
## Question (2): (20 Marks)

For the shown frame, using the stiffness method: **Neglect axial deformation**

(a) Determine the displacements at the nodes due to the given loads.

(b) Draw the bending moment diagram.

Note that $E$ and $A$ are constants, and the relative moments of inertia are given between brackets.




## Element (1): (nodes 1 & 2)

$\lambda = \cos\alpha = 1.5/4.272 = 0.351124$ and $\mu = \sin\alpha = 4/4.272 = 0.93633$

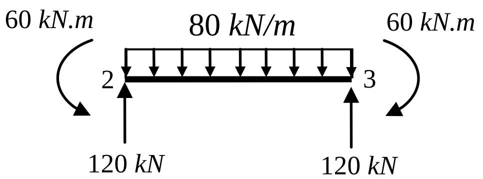
$6EI/L^2\ \mu = 6 \times EI/(4.272)^2 \times (0.93633) = 0.307835EI$

$6EI/L^2\ \lambda = 6 \times EI/(4.272)^2 \times (0.0.351124) = 0.115438EI$

$2EI/L = 2EI/4.272 = 0.468165EI$

$4EI/L = 4EI/4.272 = 0.93633EI$

$$\begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ F_{x2} \\ F_{y2} \\ M_2 \end{Bmatrix} = \left[\begin{array}{ccc|ccc} - & - & - & - & - & -0.307835EI \\ - & - & - & - & - & 0.115438EI \\ - & - & - & - & - & 0.468165EI \\ \hline - & - & - & - & - & 0.307835EI \\ - & - & - & - & - & -0.115438EI \\ - & - & - & - & - & 0.93633EI \end{array}\right] \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \theta_2 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$




**Fixed End Solution**

## Element (2): (nodes 2 & 3)

$\lambda = \cos\alpha = 1$ and $\mu = \sin\alpha = 0$

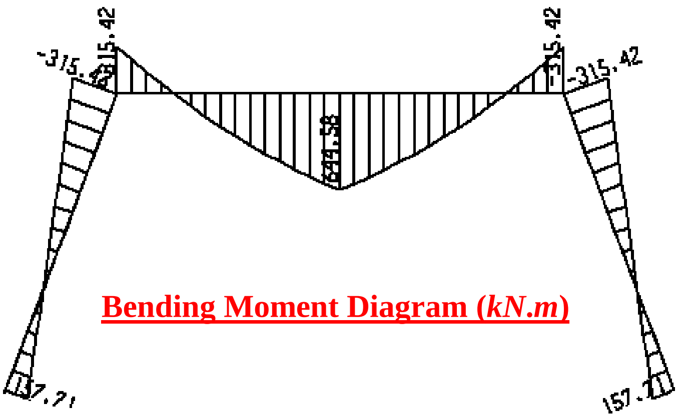
$12EI/L^3 = 12 \times E(2I)/3^3 = 0.888888EI = 8/9EI$

$6EI/L^2 = 6 \times E(2I)/3^2 = 1.333333EI = 4/3EI$

$4EI/L = 4 \times E(2I)/3 = 2.666667EI = 8/3EI$

$2EI/L = 2 \times E(2I)/3 = 1.333333EI = 4/3EI$

$$\begin{Bmatrix} F_{x2} \\ F_{y2} \\ M_2 \\ F_{x3} \\ F_{y3} \\ M_3 \end{Bmatrix} = \left[\begin{array}{ccc|ccc} - & - & 0 & - & 0 & - \\ - & - & 4/3EI & - & -8/9EI & - \\ - & - & 8/3EI & - & -4/3EI & - \\ \hline - & - & 0 & - & 0 & - \\ - & - & -4/3EI & - & 8/9EI & - \\ - & - & 4/3EI & - & -4/3EI & - \end{array}\right] \begin{Bmatrix} 0 \\ 0 \\ \theta_2 \\ 0 \\ v_3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 120 \\ 60 \\ 0 \\ 120 \\ -60 \end{Bmatrix}$$




**Bending Moment Diagram (kN.m)**

## Frame equation

$$\begin{Bmatrix} X_1 \\ Y_1 \\ M_1 \\ 0 \\ 0 \\ 0 \\ X_3 \\ -200 \\ M_3 \end{Bmatrix} = \begin{array}{c} \begin{array}{ccc} \quad 1 \quad & \quad\quad 2 \quad\quad & \quad 3 \quad \end{array} \\ \left[\begin{array}{ccc|ccc|ccc} - & - & - & - & - & -0.307835EI & - & - & - \\ - & - & - & - & - & 0.115438EI & - & - & - \\ - & - & - & - & - & 0.468165EI & - & - & - \\ \hline - & - & - & - & - & (0.307835EI + 0) & - & 0 & - \\ - & - & - & - & - & (-0.115438EI + 4/3EI) & - & -8/9EI & - \\ - & - & - & - & - & (0.93633EI + 8/3EI) & - & -4/3EI & - \\ \hline - & - & - & - & - & 0 & - & 0 & - \\ - & - & - & - & - & -4/3EI & - & 8/9EI & - \\ - & - & - & - & - & 4/3EI & - & -4/3EI & - \end{array}\right] \end{array} \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \theta_2 \\ 0 \\ v_3 \\ 0 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 120 \\ 60 \\ 0 \\ 120 \\ -60 \end{Bmatrix}$$

**From Row No. 6** → $0 = 3.603EI\ \theta_2 - 4/3EI\ v_3 + 60$

**From Row No. 8** → $-200 = -4/3EI\ \theta_2 + 8/9EI\ v_3 + 120$ → $\boxed{\theta_2 = -336.868/EI \text{ rad}}$ and $\boxed{v_3 = -865.3/EI \text{ m}}$

**From Element 1**

$M_1 = 0.468165EI(-336.868/EI) + 0 = -157.71\ kN.m$

$M_2 = 0.93633(-336.868/EI) + 0 = -315.42\ kN.m$

**From Element 2**

$M_2 = 8/3EI(-336.868/EI) - 4/3EI(-865.3) + 60 = +315.42\ kN.m$

$M_3 = 4/3EI(-336.868/EI) - 4/3EI(-865.3) - 60 = +644.58\ kN.m$

*With my best wishes*

***Dr. M. Abdel-Kader***

## Question (3): (20 Marks)

For the shown truss, using the stiffness method:

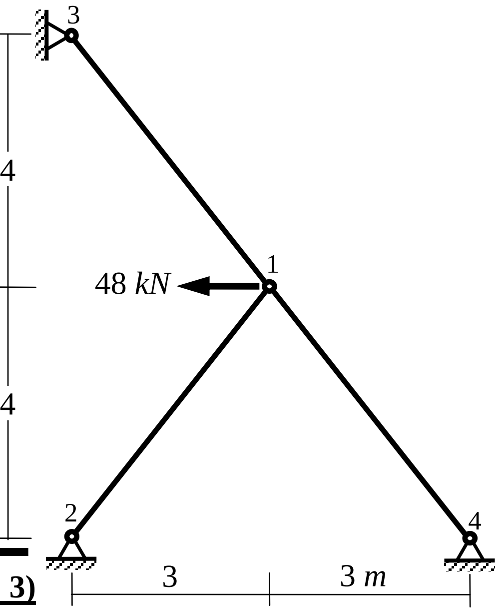
(a) Determine the displacements at the nodes due to the given loads.

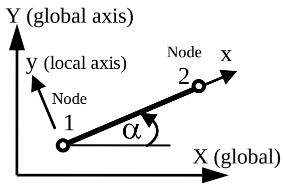
(b) Determine the reactions at the supports.

(c) Determine the forces in the members.

**Given Data:**

*EA* is constant for all members.




***Truss (Bar) element***

$$[K_e]=\begin{bmatrix} \frac{EA}{L}\lambda^2 & \frac{EA}{L}\mu\lambda & -\frac{EA}{L}\lambda^2 & -\frac{EA}{L}\mu\lambda \\ \frac{EA}{L}\mu\lambda & \frac{EA}{L}\mu^2 & -\frac{EA}{L}\mu\lambda & -\frac{EA}{L}\mu^2 \\ -\frac{EA}{L}\lambda^2 & -\frac{EA}{L}\mu\lambda & \frac{EA}{L}\lambda^2 & \frac{EA}{L}\mu\lambda \\ -\frac{EA}{L}\mu\lambda & -\frac{EA}{L}\mu^2 & \frac{EA}{L}\mu\lambda & \frac{EA}{L}\mu^2 \end{bmatrix}$$

Where, $\lambda = \cos\alpha$ and $\mu = \sin\alpha$

## Element (1): (nodes 1 & 2)

$\lambda = \cos\alpha = -0.6$ and $\mu = \sin\alpha = -0.8$

$EA/L = EA/5 = 0.2\,EA$

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x2} \\ F_{y2} \end{Bmatrix} = \begin{bmatrix} 0.072EA & 0.096EA & - & - \\ 0.096EA & 0.128EA & - & - \\ -0.072EA & -0.096EA & - & - \\ -0.096EA & -0.128EA & - & - \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \end{Bmatrix}$$

## Element (2): (nodes 1 & 3)

$\lambda = -0.6$ and $\mu = 0.8$

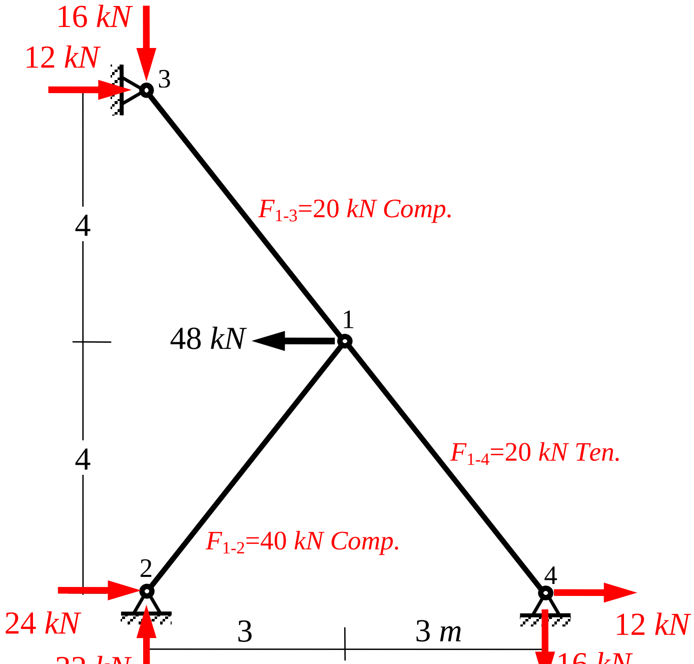
$EA/L = EA/5 = 0.2\,EA$

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x3} \\ F_{y3} \end{Bmatrix} = \begin{bmatrix} 0.072EA & -0.096EA & - & - \\ -0.096EA & 0.128EA & - & - \\ -0.072EA & 0.096EA & - & - \\ 0.096EA & -0.128EA & - & - \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \end{Bmatrix}$$

## Element (3): (nodes 1 & 4)

$\lambda = 0.6$ and $\mu = -0.8$ $\quad EA/L = EA/5 = 0.2\,EA$

$$\begin{Bmatrix} F_{x1} \\ F_{y1} \\ F_{x4} \\ F_{y4} \end{Bmatrix} = \begin{bmatrix} 0.072EA & -0.096EA & - & - \\ -0.096EA & 0.128EA & - & - \\ -0.072EA & 0.096EA & - & - \\ 0.096EA & -0.128EA & - & - \end{bmatrix} \begin{Bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \end{Bmatrix}$$




***Final Reactions***

## Truss equation

$$\begin{array}{c} \\ \begin{Bmatrix} -48 \\ 0 \\ X_2 \\ Y_2 \\ X_3 \\ Y_3 \\ X_4 \\ Y_4 \end{Bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} \quad 1 \quad\quad & \quad\quad 2 \quad & \quad 3 \quad & \quad 4 \quad \end{array} \\ \begin{bmatrix} 0.216EA & -0.096EA & - & - & - & - & - & - \\ -0.096EA & 0.384EA & - & - & - & - & - & - \\ -0.072EA & -0.096EA & - & - & 0 & 0 & 0 & 0 \\ -0.096EA & -0.128EA & - & - & 0 & 0 & 0 & 0 \\ -0.072EA & 0.096EA & 0 & 0 & - & - & 0 & 0 \\ 0.096EA & -0.128EA & 0 & 0 & - & - & 0 & 0 \\ -0.072EA & 0.096EA & 0 & 0 & 0 & 0 & - & - \\ 0.096EA & -0.128EA & 0 & 0 & 0 & 0 & - & - \end{bmatrix} \end{array} \begin{array}{c} \\ \begin{Bmatrix} u_1 \\ v_1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \end{array}$$

**From Row No. 1** $\rightarrow$ $-48 = 0.216EA\,u_1 - 0.096EA\,v_1$

**From Row No. 2** $\rightarrow$ $0 = -0.096EA\,u_2 + 0.384EA\,v_2$ $\rightarrow$

$\boxed{u_1 = -250/EA\ m}$ $\leftarrow$

$\boxed{v_1 = -62.5/EA\ m}$ $\downarrow$

**From Row No. 3** $\rightarrow$ $X_2 = -0.072EA\,(-250/EA) - 0.096\,EA\,(-62.5/EA) = 24\ kN$ $\quad \boxed{X_2 = 24\ kN \rightarrow}$

**From Row No. 4** $\rightarrow$ $Y_2 = -0.096EA\,(-250/EA) - 0.128\,EA\,(-62.5/EA) = 32\ kN$ $\quad \boxed{Y_2 = 32\ kN \uparrow}$

**From Row No. 5** $\rightarrow$ $X_3 = -0.072EA\,(-250/EA) + 0.096\,EA\,(-62.5/EA) = 12\ kN$ $\quad \boxed{X_3 = 12\ kN \rightarrow}$

**From Row No. 6** $\rightarrow$ $Y_3 = 0.096EA\,(-250/EA) - 0.128\,EA\,(-62.5/EA) = -16\ kN$ $\quad \boxed{Y_3 = 16\ kN \downarrow}$

**From Row No. 7** $\rightarrow$ $X_4 = -0.072EA\,(-250/EA) + 0.096\,EA\,(-62.5/EA) = 12\ kN$ $\quad \boxed{X_4 = 12\ kN \rightarrow}$

**From Row No. 8** $\rightarrow$ $Y_4 = 0.096EA\,(-250/EA) - 0.128\,EA\,(-62.5/EA) = -16\ kN$ $\quad \boxed{Y_4 = 16\ kN \downarrow}$

*With my best wishes*

***Dr. M. Abdel-Kader***